The Cosmological Neutrino Background

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Pillars of the Cosmological Model

- Hubble law

$d_L = (1+z)\ x$, x comoving distance ✔

- CMB

black body distribution T= 2.275 °K ✔

- BBN

light nuclei forms at T =MeV – 10 keV ✔

- Cosmic Neutrino Background (CNB) direct measurement ??

CNB Relic neutrino production and decoupling

$1\ \mathrm{MeV} \leq \mathbf{T} \leq m_\mu$

$$T_\nu = T_e = T_\gamma$$

$$\nu_\alpha \nu_\beta \leftrightarrow \nu_\alpha \nu_\beta$$

$$\nu_\alpha \bar{\nu}_\beta \leftrightarrow \nu_\alpha \bar{\nu}_\beta$$

$$\nu_\alpha e^- \leftrightarrow \nu_\alpha e^-$$

$$\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$$

$$\mathcal{L}_{\mathrm{SM}} = -2\sqrt{2} G_F \left\{ \left(\bar{\nu}_e \gamma^\mu L \nu_e \right) \left(\bar{e} \gamma_\mu L e \right) + \sum_{P,\alpha} g_P \left(\bar{\nu}_\alpha \gamma^\mu L \nu_\alpha \right) \left(\bar{e} \gamma_\mu P e \right) \right\}$$

$$P = L,\ R = (1 \mp \gamma_5)/2$$

$$g_L = -\tfrac{1}{2} + \sin^2\theta_W \text{ and } g_R = \sin^2\theta_W$$

Neutrino decoupling

As the Universe expands, particle densities are diluted and temperatures fall. Weak interactions become ineffective to keep neutrinos in good thermal contact with the e.m. plasma

Rough, but quite accurate estimate of the decoupling temperature

Rate of weak processes ~ Hubble expansion rate

$$\Gamma_w \approx \sigma_w |\mathrm{v}| n \,,\; H^2 = \frac{8\pi\rho_R}{3M_p^2} \rightarrow G_F^2 T^5 \approx \sqrt{\frac{8\pi\rho_R}{3M_p^2}} \rightarrow T_{dec}^{\nu} \approx 1\, MeV$$

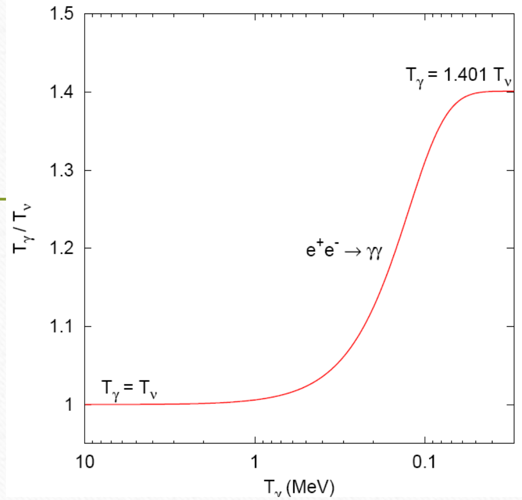
Since ν_e have both CC and NC interactions with $e^{\pm}$

$T_{dec}(\nu_e) \sim 2$ MeV

$T_{dec}(\nu_{\mu,\tau}) \sim 3$ MeV

Neutrino and Photon (CMB) temperatures

$$f_\nu(p,T) = \frac{1}{e^{p/T_\nu}+1}$$

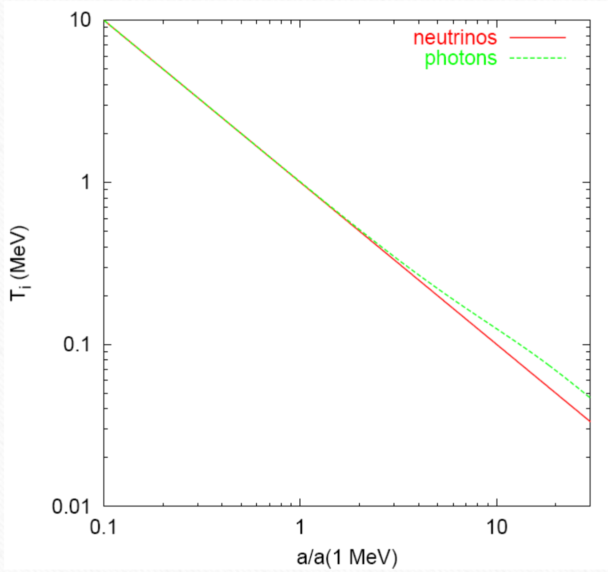


At $T \sim m_e$, electron-positron pairs annihilate

$$e^+e^- \to \gamma\gamma$$

heating photons but not the decoupled neutrinos

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$



Neutrinos decoupled at T~MeV, keeping a spectrum as that of a relativistic species

$$f_\nu(p,T) = \frac{1}{e^{p/T_\nu}+1}$$

$$n_\nu = \int \frac{d^3p}{(2\pi)^3} f_\nu(p,T_\nu) = \frac{3}{11} n_\gamma = \frac{6\zeta(3)}{11\pi^2} T^3_{CMB}$$

$$\rho_{\nu_i} = \int \sqrt{p^2+m^2_{\nu_i}} \frac{d^3p}{(2\pi)^3} f_\nu(p,T_\nu) \rightarrow \begin{cases} \frac{7\pi^2}{120}\left(\frac{4}{11}\right)^{4/3} T^4_{CMB} & \text{Massless} \\ m_{\nu_i} n_\nu & \text{Massive } m_\nu >> T \end{cases}$$

$$\Omega_\nu h^2 = 1.7 \times 10^{-5}$$

$$\Omega_\nu h^2 = \frac{\sum_i m_i}{94.1\ \text{eV}}$$

De Broglie wavelength today $\lambda \approx mm$ coherent scattering on targets?

Number density today $112\ \text{cm}^{-3}$ per flavour

N_{eff}

$$\rho_R = \rho_\gamma + \rho_v + \rho_x = \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N^{eff}{}_v\right)\rho_\gamma$$

CNB details

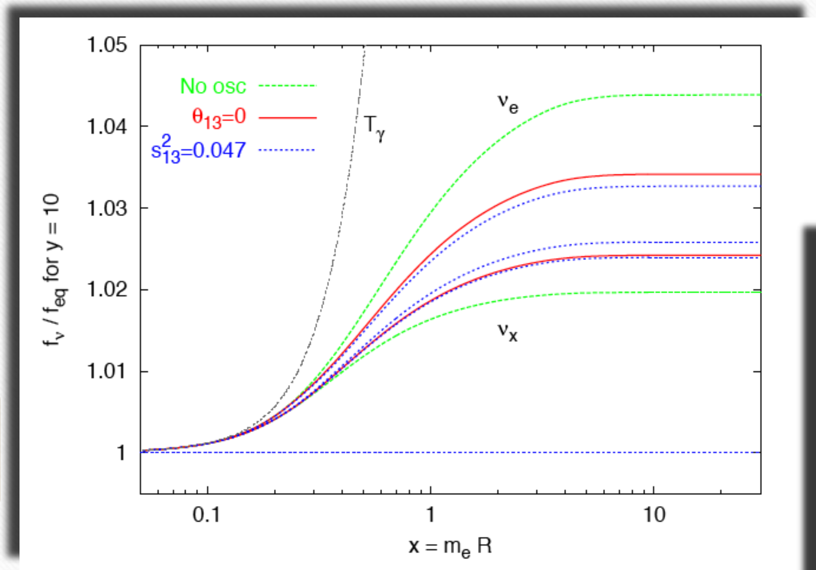
At $T \sim m_e$, e^+e^- pairs annihilate heating photons

$$e^+e^- \to \gamma\gamma$$

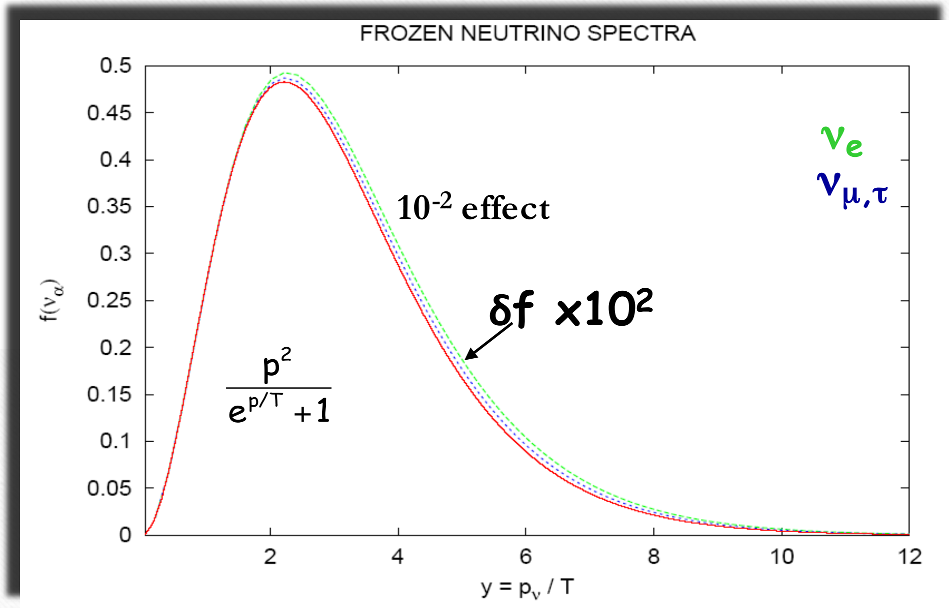
… and neutrinos. Non thermal features in ν distribution (small effect). Oscillations slightly modify the result

$$f_\nu = f_{FD}(p, T_\nu)[1+\delta f(p)]$$

$$\left(i\partial_t - Hp\partial_p\right)\rho = \left[\frac{M^2}{p} - \frac{8\sqrt{2}G_F}{m_W^{\;2}}E, \rho\right] + C(\rho)$$



No osc
θ13=0
s²13=0.047
Tγ
νe
νx
fν / feq for y = 10
x = me R



FROZEN NEUTRINO SPECTRA
νe
νμ,τ
10-2 effect
δf ×10²
p² / (e^{p/T} +1)
f(να)
y = pν / T

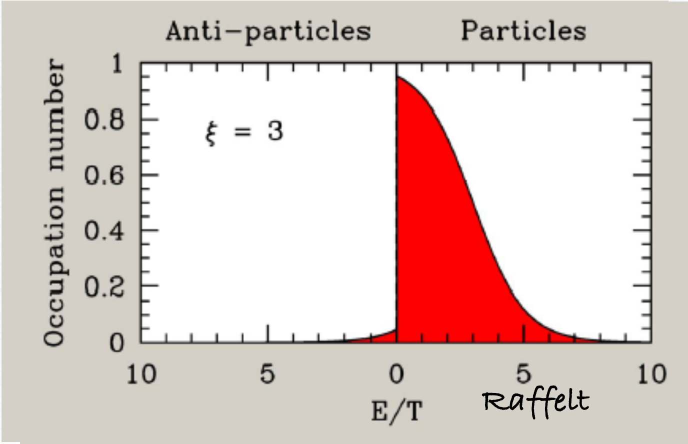
Results

	$T^{\gamma}_{fin} / T^{\gamma}_{0}$	$\delta\rho_{\nu_e}$(%)	$\delta\rho_{\nu_\mu}$ (%)	$\delta\rho_{\nu_\tau}$(%)	N_{eff}
Instantaneous decoupling	1.40102	0	0	0	3
SM	1.3978	0.94	0.43	0.43	3.046
+3ν mixing (θ_{13}=0)	1.3978	0.73	0.52	0.52	3.046
+3ν mixing ($\sin^2\theta_{13}$=0.047)	1.3978	0.70	0.56	0.52	3.046

Dolgov, Hansen & Semikoz, NPB 503 (1997) 426
G.M. et al, PLB 534 (2002) 8
G.M. et al, NPB 729 (2005) 221

CνB details

Fermi-Dirac spectrum with temperature T and chemical potential $\mu_\nu = \xi_\nu T_\nu$

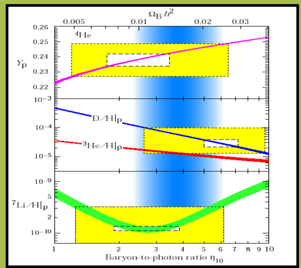
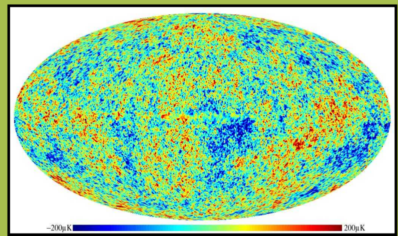
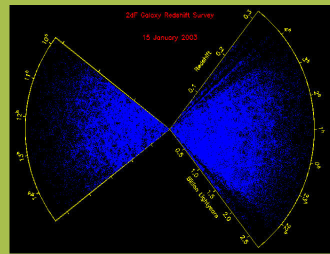


$$n_\nu \neq n_{\bar{\nu}}$$

$$L_\nu = \frac{n_\nu - n_{\bar{\nu}}}{n_\gamma} = \frac{1}{12\zeta(3)}\left(\frac{T_\nu}{T_\gamma}\right)^3 \left[\pi^2 \xi_\nu + \xi_\nu^3\right]$$

$$\Delta\rho_\nu = \frac{15}{7}\left[2\left(\frac{\xi_\nu}{\pi}\right)^2 + \left(\frac{\xi_\nu}{\pi}\right)^4\right]$$

CNB indirect evidences

		
Primordial Nucleosynthesis BBN	Cosmic Microwave Background CMB	Formation of Large Scale Structures LSS
T ~ MeV	T < eV	
flavor dependent	Flavor blind	

BBN: almost seventy years after αβγ seminal paper(Alpher, Bethe & Gamow 1948)

- Theory reasonably under control (per mille level for ^{4}He (neutron lifetime), 1-2 % for ^{2}H);
- Increased precision in nuclear reaction cross sections at low energy (underground lab's);
- $\Omega_b h^2$ measured by WMAP/Planck with high precision;
- Decreasingly precise data (^{4}He, but see later), ^{7}Li not understood, ^{2}H fixes $\Omega_b h^2$ value in good agreement with CMB data.

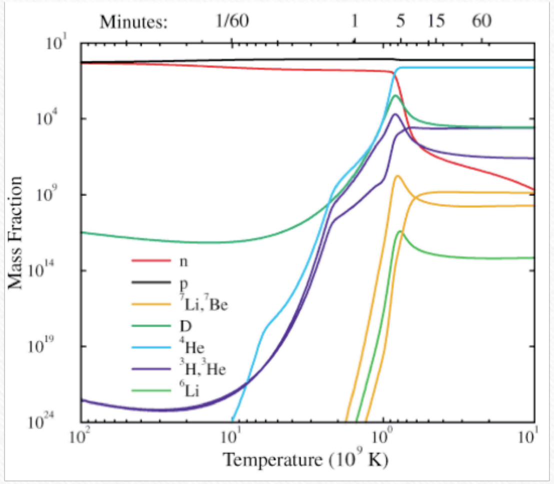
THEORY

weak rate freeze out (1 MeV);
^{2}H forms at T~0.08 MeV;
nuclear chain;

Z \ N	0	1	2	3	4	5	6	7	8
0		n							
1	H	^{2}H	^{3}H						
2		^{3}He	^{4}He						
3				^{6}Li	^{7}Li	^{8}Li			
4				^{7}Be		^{9}Be			
5				^{8}B		^{10}B	^{11}B	^{12}B	
6						^{11}C	^{12}C	^{13}C	^{14}C
7						^{12}N	^{13}N	^{14}N	^{15}N
8							^{14}O	^{15}O	^{16}O

Iocco et al, Phys Rept. 472, 1 (2009)

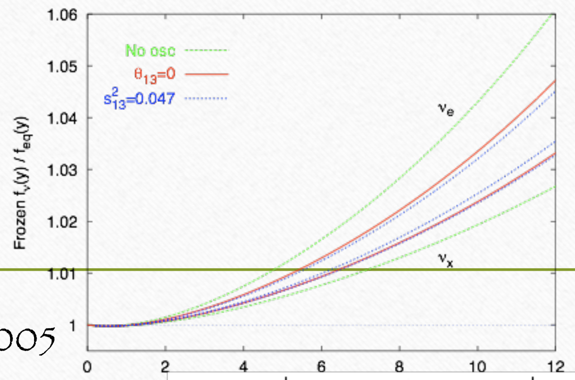
Public numerical codes:Kawano, PArthENoPE, PRIMAT
private numerical codes: many...



PArthENoPE
Public Algorithm Evaluating Nucleosynthesis of Primordial Elements

Weak rates:

radiative corrections $O(\alpha)$
finite nucleon mass $O(T/M_N)$
plasma effects $O(\alpha T/m_e)$
neutrino decoupling $O(G_F^2 T^3 m_{Pl})$



$N_{eff}=3.046$

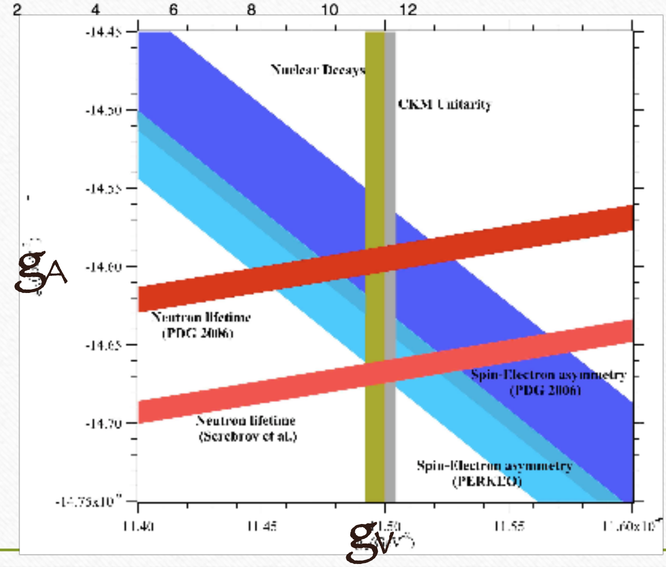
G.M. et al 2005

Main uncertainty: neutron lifetime
τ_n= 885.6 ± 0.8 sec (old PDG mean)
τ_n=878.5 ± 0.8 sec (Serebrov et al 2005)

Presently:

τ_n=880.3 ± 1.1 sec

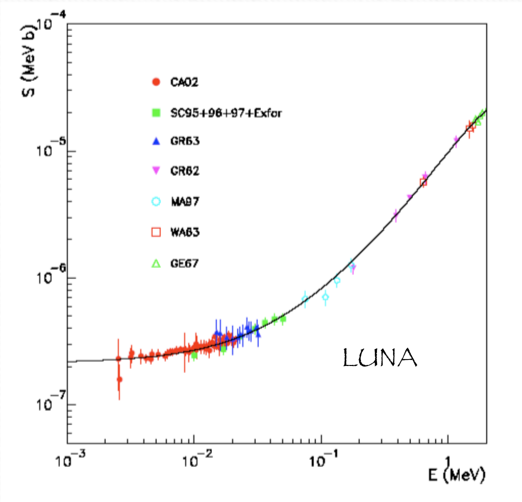
^{4}He mass fraction Y_P linearly increases with τ_n: 0.246 - 0.249



Nico & Snow 2006

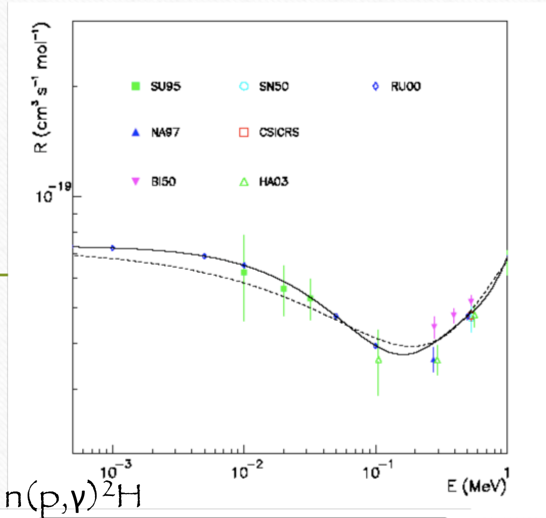
Nuclear rates:

main input from experiments
low energy range (10^2 KeV)
major improvement: underground measurements (e.g. LUNA at LNGS)

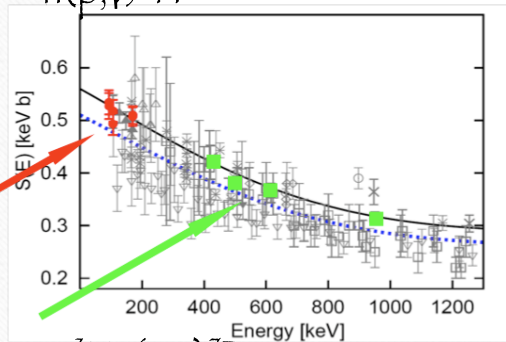


$^2H(p,\gamma)^3He$

Rupak



$n(p,\gamma)^2H$



LUNA

Weitzmann Inst.

$^3He(\alpha,\gamma)^7Be$

ERNA: S(0)=0.57±0.04 KeV b   Di Leva et al 2010

DATA

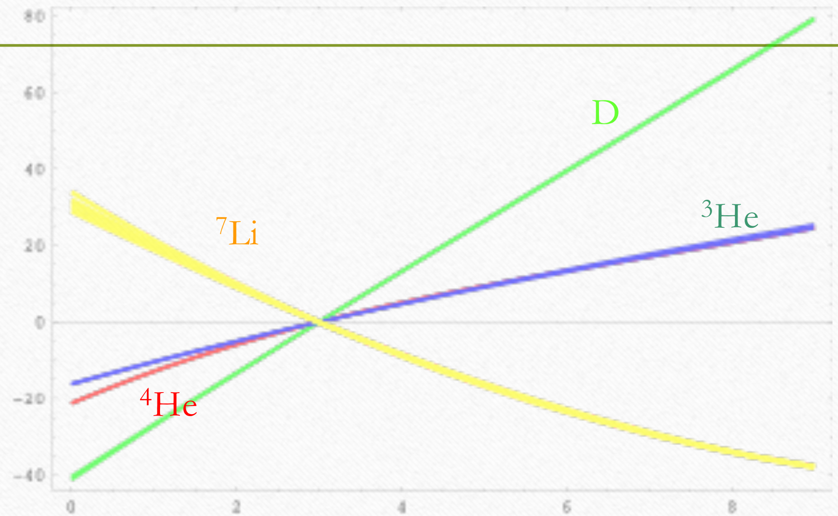
The quest for primordiality

- Observations in systems negligibly contaminated by stellar evolution (e.g. high redshift);
- Careful account for galactic chemical evolution.

Effect of neutrinos on BBN

1. N_{eff} fixes the expansion rate during BBN

$$\rho_R = \rho_\gamma + \rho_\nu + \rho_x = \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N^{eff}{}_\nu\right)\rho_\gamma$$



2. Direct effect of electron neutrinos and antineutrinos on the n-p reactions

$$\nu_e + n \longleftrightarrow p + e^- \qquad e^+ + n \longleftrightarrow p + \bar{\nu}_e$$

Neutrino-antineutrino asymmetry ($\xi=\mu/T_\nu$, $E_F(\xi)$) strongly constrained by Big Bang Nucleosynthesis

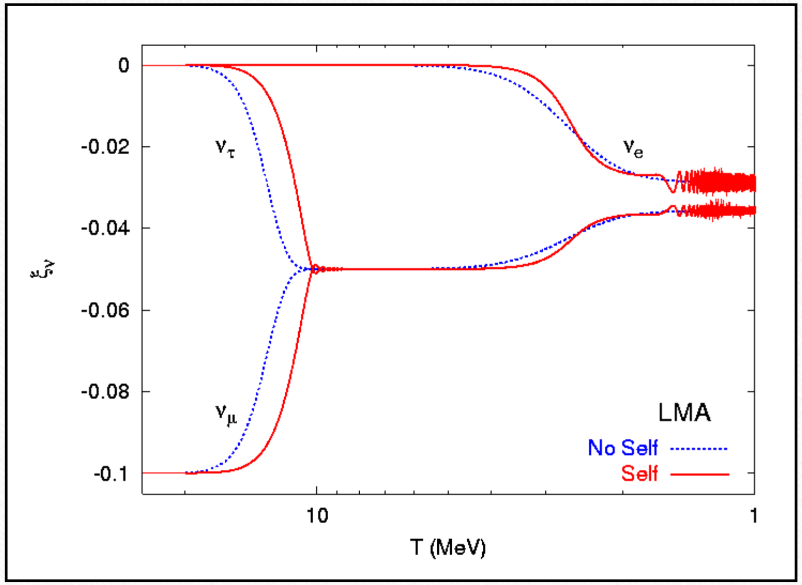
1) **chemical potentials contribute to neutrino energy density**

$$\rho_\nu = \frac{7\pi^2}{120}\left(3 + \sum_i \left(\frac{30\xi_i^2}{7\pi^2} + \frac{15\xi_i^4}{7\pi^4}\right) + ...\right)T_\nu^4$$

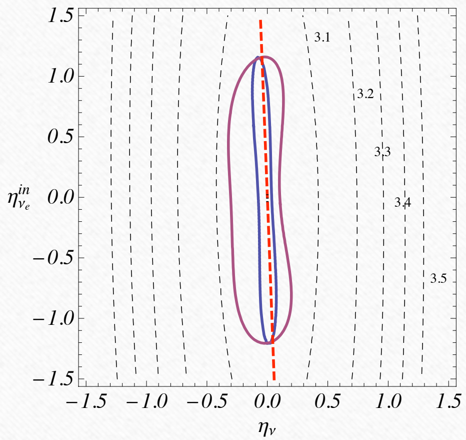
2) **a positive electron neutrino chemical potential (more neutrinos than antineutrinos) favour n $\Rightarrow$ p processes with respect to p n processes.**

Change the ^{4}He abundance!

Though different neutrino flavor may have different chemical potentials, they however mix under oscillations



Likelihood contours 68 & 95 c.l.

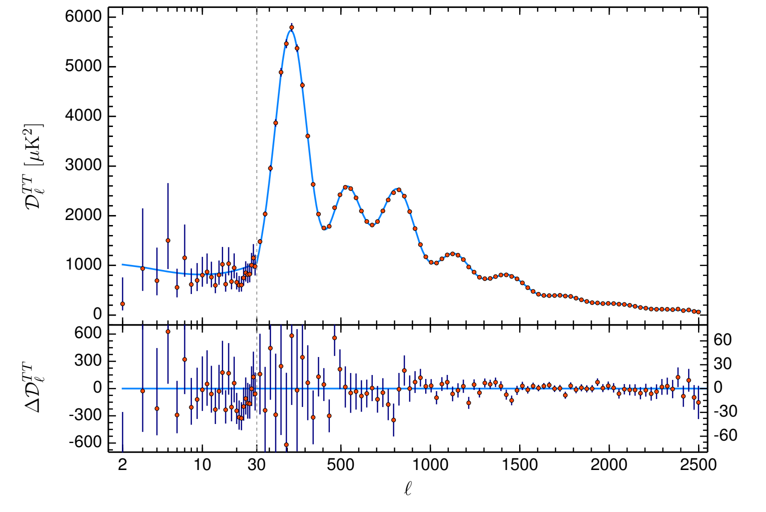


ξ very small!

The CMB

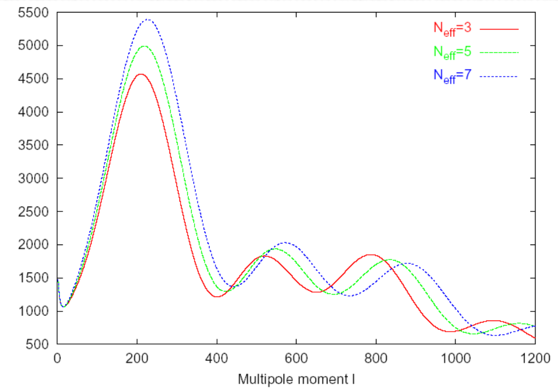
Anisotropies contain so much information abut the cosmological model!

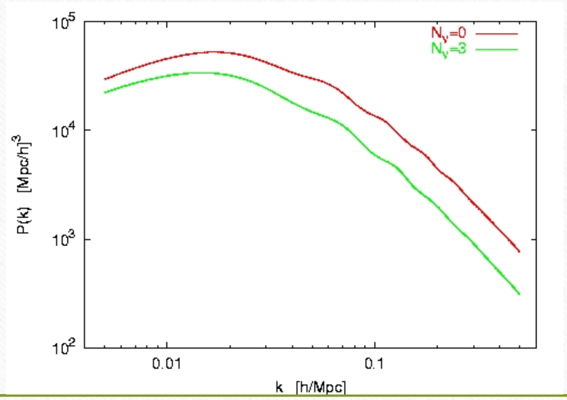
Planck 2018



Effect of CNB on CMB and LSS

Mean effect (Sachs-Wolfe, M-R equality)+ perturbations





Perturbations

Acoustic peak amd damping tail: N_{eff}
Lensing potential on CMB: m_ν larger expansion rate suppresses clustering

Large Scale Structure: suppression at small scales $k > 0.1$ h Mpc^{-1}

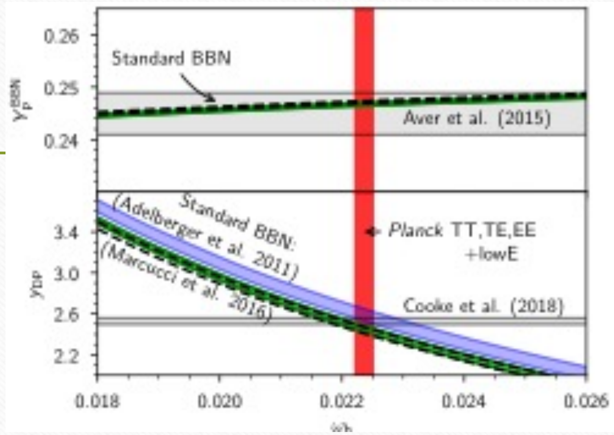
CMB+LSS: allowed ranges for N_{eff}

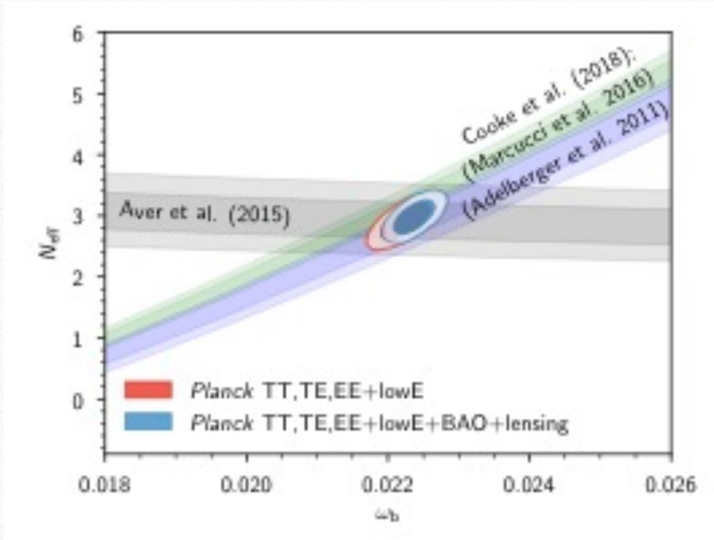
Set of parameters: ($\Omega_b h^2$, $\Omega_{cdm} h^2$, h , n_s , A , b , N_{eff})

- DATA: Planck , Flat Models

$N_{\rm eff} = 3.11^{+0.44}_{-0.43}$ (95 %, TT+lowE+lensing+BAO);

$N_{\rm eff} = 2.99^{+0.34}_{-0.33}$ (95 %, TT,TE,EE+lowE+lensing +BAO).


Standard BBN
Aver et al. (2015)
Standard BBN:
(Adelberger et al. 2011)
(Marcucci et al. 2016)
Planck TT,TE,EE
+lowE
Cooke et al. (2018)
0.018
0.020
0.022
0.024
0.026


Cooke et al. (2018);
(Marcucci et al. 2016)
(Adelberger et al. 2011)
Aver et al. (2015)
Planck TT,TE,EE+lowE
Planck TT,TE,EE+lowE+BAO+lensing
N_{eff}
0.018
0.020
0.022
0.024
0.026

Neutrino masses

Terrestrial bounds

ν_e <2 eV (^{3}H decay)

ν_μ <0.19 MeV (pion decays)

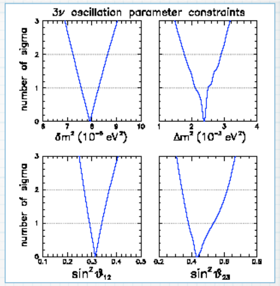
ν_τ <18.2 MeV (τ decays)

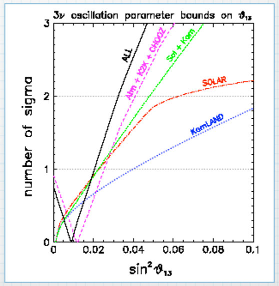
Oscillation Parameters

$$\delta m^2 = 7.92(1 \pm 0.09) \times 10^{-5}\ \mathrm{eV}^2$$
$$\sin^2\theta_{12} = 0.314(1^{+0.18}_{-0.15})$$

$$\Delta m^2 = 2.6(1^{+0.14}_{-0.15}) \times 10^{-3}\ \mathrm{eV}^2$$
$$\sin^2\theta_{23} = 0.45(1^{+0.35}_{-0.20})$$

$$\sin^2\theta_{13} = 0.8(1^{+2.3}_{-0.8}) \times 10^{-2}$$





CMB Planck 2018

$$\sum m_\nu < 0.44\ \mathrm{eV} \quad (95\,\%,\ \mathrm{TT{+}lowE{+}lensing}),$$
$$\sum m_\nu < 0.24\ \mathrm{eV} \quad (95\,\%,\ \mathrm{TT,TE,EE{+}lowE{+}lensing}).$$

Clustering and ν local density

Massive neutrinos cluster on CDM and baryonic structures. The local density at Earth (8 kpc away from the galactic center) is expected to be larger than 56 cm^{-3}

$$\frac{\partial f_i}{\partial \tau} + \frac{\boldsymbol{p}}{am_i} \cdot \frac{\partial f_i}{\partial \boldsymbol{x}} - am_i \nabla \phi \cdot \frac{\partial f_i}{\partial \boldsymbol{p}} = 0,$$

$$\nabla^2 \phi = 4\pi G a^2 \sum_i \overline{\rho}_i(\tau) \delta_i(\boldsymbol{x}, \tau),$$

$$\delta_i(\boldsymbol{x}, \tau) \equiv \frac{\rho_i(\boldsymbol{x}, \tau)}{\overline{\rho}_i(\tau)} - 1, \qquad \rho_i(\boldsymbol{x}, \tau) = \frac{m_i}{a^3} \int d^3p\, f_i(\boldsymbol{x}, \boldsymbol{p}, \tau),$$

Neutrinos accrete when their velocity becomes comparable with protocluster velocity dispersion ($z<2$)

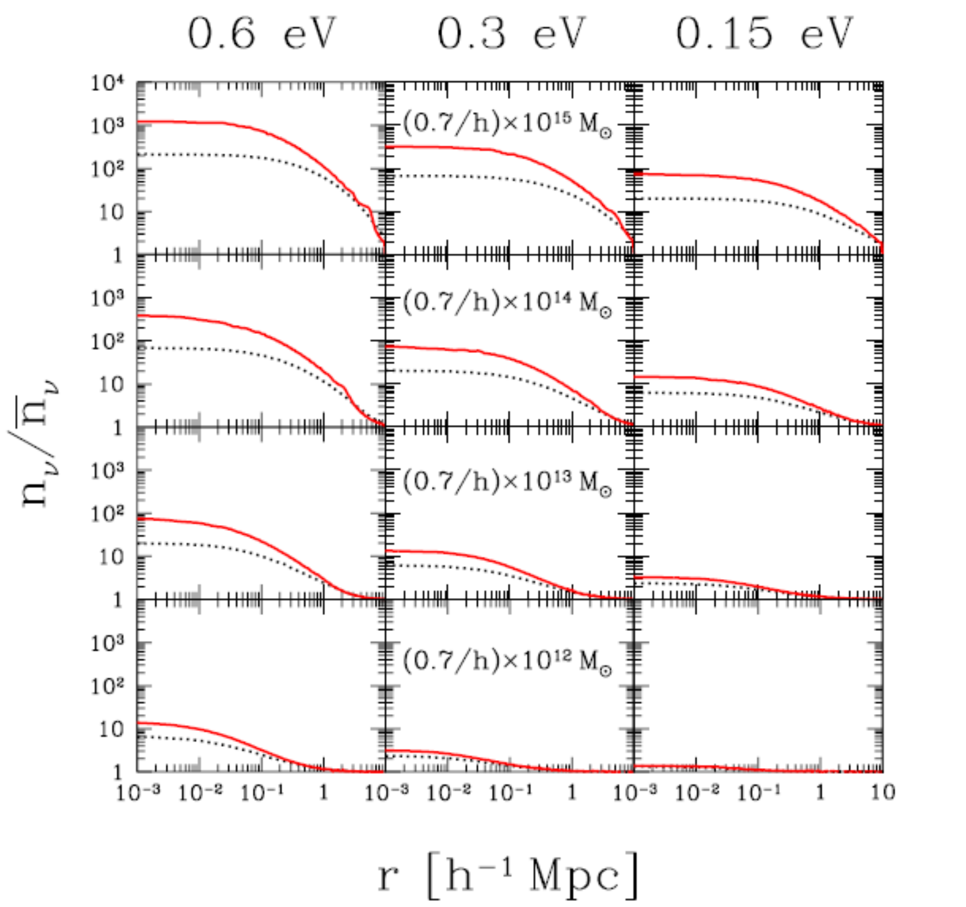
Usual assumption: Halo profile governed by CDM only

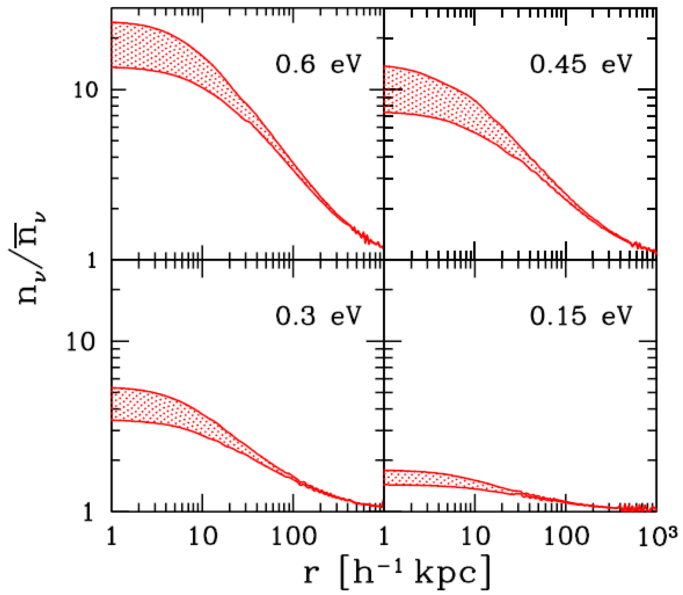
NFW universal profile

$$\rho_{\mathrm{halo}}(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2},$$

A. Ringwald and Y. Wong 2004

N-1-body simulations



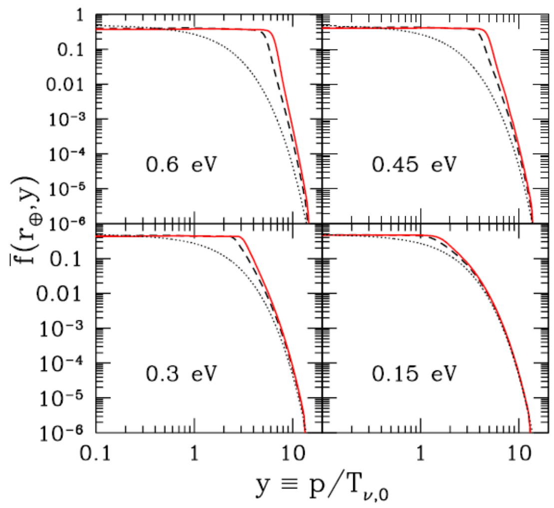


Milky Way

Top curve: NFW Bottom curve: static present MW matter profile

A. Ringwald and Y. Wong 2004

N-1-body simulations



Marcello…..

Backup slides

CNB direct detection

CNB: very low energy, difficult to measure directly by ν-scattering

1. Large De Broglie wavelength λ~0.1 cm

   Coherent scattering over nuclei (or macroscopic domain)

   Wind force on a test body,

Cross section

$\sigma_{\nu N}$ ~10^{-56} (m_ν/eV)^2 cm^2 non relativistic

$\sigma_{\nu N}$ ~10^{-63} (T_ν/eV)^2 cm^2 relativistic

acceleration

$n_\nu \, \beta \, NA/A \, \sigma_{\nu N} \, dp \sim (100/A) 10^{-51} (m_\nu/\mathrm{eV}) \, \mathrm{cm\, s^{-2}}$

Today: Cavendish torsion balances can test acceleration as small as 10^{-13} cm s^{-2} !!

2. Accelerators:

Too small even at LHC or beyond !

3. Effects linear in G_F:

No go theorem (Cabibbo & Maiani, Langacker et al) effect vanishes if

static source - background interaction

Homogeneous ν flux on the target scale

Stodolski effect: polarized electron target experiences a tourque due to helicity energy slpitting in presence of a polarized (asymmetry) neutrino wind

$dE \sim g_A \, \vec{\sigma} \cdot \vec{\beta}(n_\nu - n_{\bar{\nu}})$

A '62 paper by S. Weinberg and ν chemical potential

PHYSICAL REVIEW VOLUME 128, NUMBER 3 NOVEMBER 1, 1962

Universal Neutrino Degeneracy

STEVEN WEINBERG*
Imperial College of Science and Technology, London, England
(Received March 22, 1962)

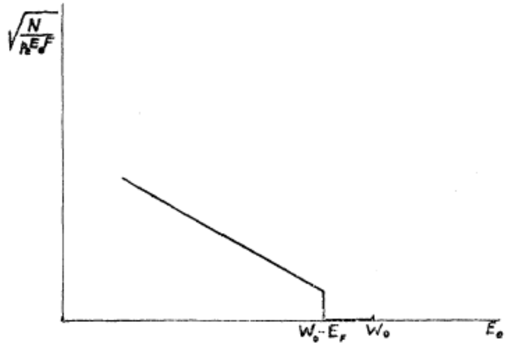


FIG. 1. Shape of the upper end of an allowed Kurie plot to be expected in a β^+ decay if neutrinos are degenerate up to energy E_F, or in a β^- decay if antineutrinos are degenerate.

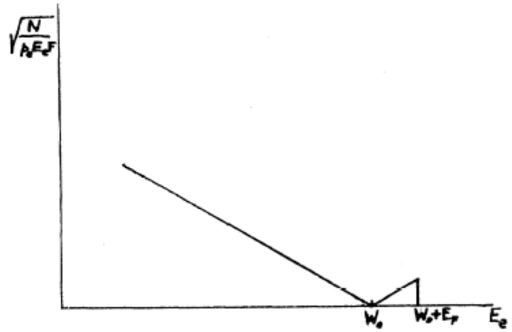
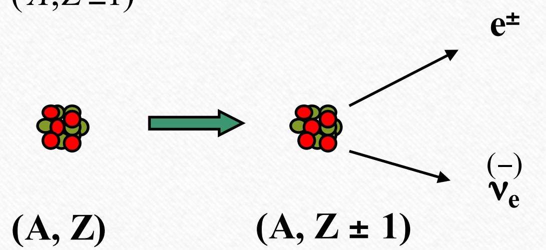


FIG. 2. Shape of the upper end of an allowed Kurie plot to be expected in a β^- decay if neutrinos are degenerate up to energy E_F, or in a β^+ decay if antineutrinos are degenerate.

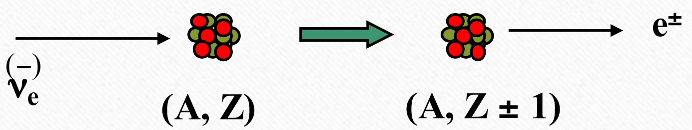
$$N_{(A,Z)} \rightarrow N'_{(A,Z\pm1)}\, e^{\pm}\, \overset{(-)}{\nu}_e$$

$$\overset{(-)}{\nu}_e\, N_{(A,Z)} \rightarrow N'_{(A,Z\pm1)}\, e^{\pm}$$

Beta decay



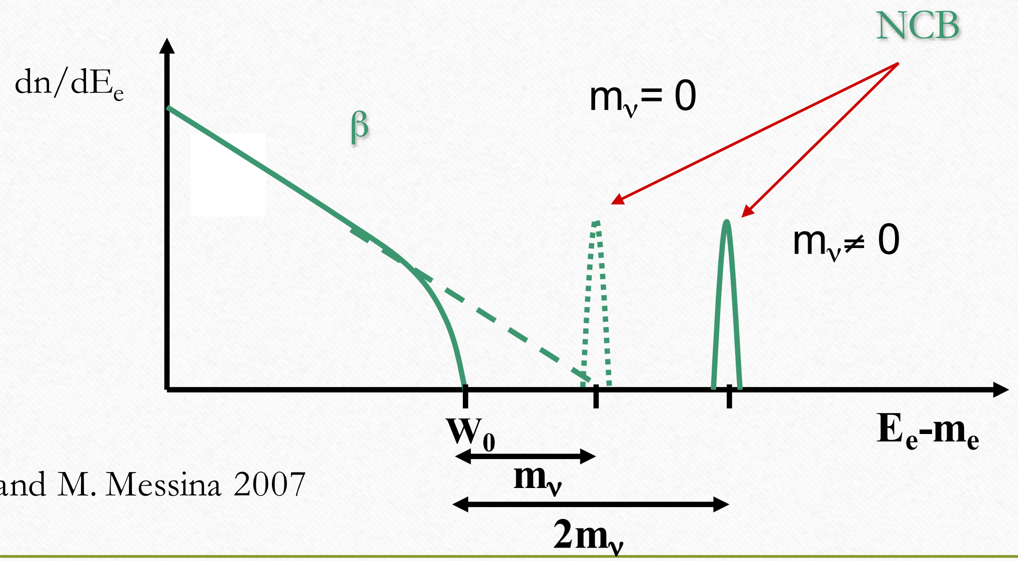
Neutrino Capture on a
Beta Decaying Nucleus
(NCβ)



Weinberg: if neutrinos are degenerate we could observe structures around the beta decaying nuclei endpoint Q

ν's are NOT degenerate but are massive!

2 m_ν gap in electron spectrum around Q



A. Cocco, G.M. and M. Messina 2007

Issues:

1. Rates

$$\lambda_\nu = \int \sigma_{\rm NCB} v_\nu \, f(p_\nu) \, \frac{d^3 p_\nu}{(2\pi)^3} \, , \quad = \frac{G_\beta^2}{2\pi^3} \int_{W_o + 2m_\nu}^{\infty} p_e E_e F(Z, E_e) C(E_e, p_\nu)_\nu \cdot E_\nu p_\nu \, f(p_\nu) \, dE_e \, ,$$

$$\lambda_\beta = \frac{G_\beta^2}{2\pi^3} \int_{m_e}^{W_o} p_e E_e F(Z, E_e) C(E_e, p_\nu)_\beta E_\nu p_\nu \, dE_e \, ,$$

Nuclear form factors (shape factors) uncertainties: use beta observables

$$\mathcal{A} = \int_{m_e}^{W_o} \frac{C(E'_e, p'_\nu)_\beta}{C(E_e, p_\nu)_\nu} \frac{p'_e}{p_e} \frac{E'_e}{E_e} \frac{F(E'_e, Z)}{F(E_e, Z)} E'_\nu p'_\nu dE'_e$$

$$\sigma_{\rm NCB} v_\nu = \frac{2\pi^2 \ln 2}{\mathcal{A} \, t_{1/2}}$$

Cross sections times v_ν as high as 10^{-41} cm^2 c

Table 1. The product $\sigma_{\rm NCB}(v_\nu/c)$ for the best known superallowed $0^+ \rightarrow 0^+$ transitions. Numerical values for Q_β and partial half-lifes are taken from [33]. The value of f is calculated adopting the parametrization of the Fermi function of [28].

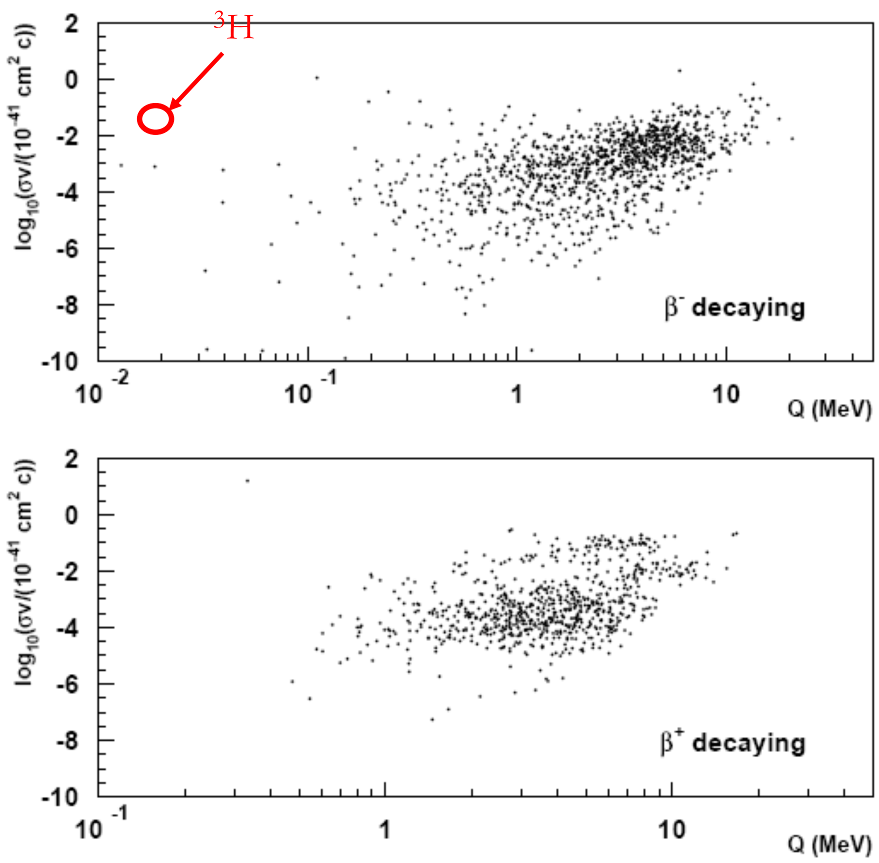
Isotope	Q_β (keV)	Half-life (sec)	$\sigma_{\rm NCB}(v_\nu/c)$ (10^{-41} cm^2)
^{10}C	885.87	1320.99	5.36×10^{-3}
^{14}O	1891.8	71.152	1.49×10^{-2}
^{26m}Al	3210.55	6.3502	3.54×10^{-2}
^{34}Cl	4469.78	1.5280	5.90×10^{-2}
^{38m}K	5022.4	0.92512	7.03×10^{-2}
^{42}Sc	5403.63	0.68143	7.76×10^{-2}
^{46}V	6028.71	0.42299	9.17×10^{-2}
^{50}Mn	6610.43	0.28371	1.05×10^{-1}
^{54}Co	7220.6	0.19350	1.20×10^{-1}

A. Cocco, G.M. and M. Messina 2007

Table 2. Beta decaying nuclei that present the largest product of $\sigma_{\rm NCB}(v_\nu/c) \cdot t_{1/2}$ for low neutrino momentum and have a $\beta^\pm$ decay branching fraction larger than 80%.

Isotope	Decay	Q_β (keV)	Half-life (sec)	$\sigma_{\rm NCB}(v_\nu/c)$ (10^{-41} cm^2)
^{3}H	β^-	18.591	3.8878×10^{8}	7.84×10^{-4}
^{63}Ni	β^-	66.945	3.1588×10^{9}	1.38×10^{-6}
^{93}Zr	β^-	60.63	4.952×10^{13}	2.39×10^{-10}
^{106}Ru	β^-	39.4	3.2278×10^{7}	5.88×10^{-4}
^{107}Pd	β^-	33	2.0512×10^{14}	2.58×10^{-10}
^{187}Re	β^-	2.64	1.3727×10^{18}	4.32×10^{-11}
^{11}C	β^+	960.2	1.226×10^{3}	4.66×10^{-3}
^{13}N	β^+	1198.5	5.99×10^{2}	5.3×10^{-3}
^{15}O	β^+	1732	1.224×10^{2}	9.75×10^{-3}
^{18}F	β^+	633.5	6.809×10^{3}	2.63×10^{-3}
^{22}Na	β^+	545.6	9.07×10^{7}	3.04×10^{-7}
^{45}Ti	β^+	1040.4	1.307×10^{4}	3.87×10^{-4}

Beta decaying nuclei having BR(β±) > 5 % selected from 14543 decays listed in the ENSDF database

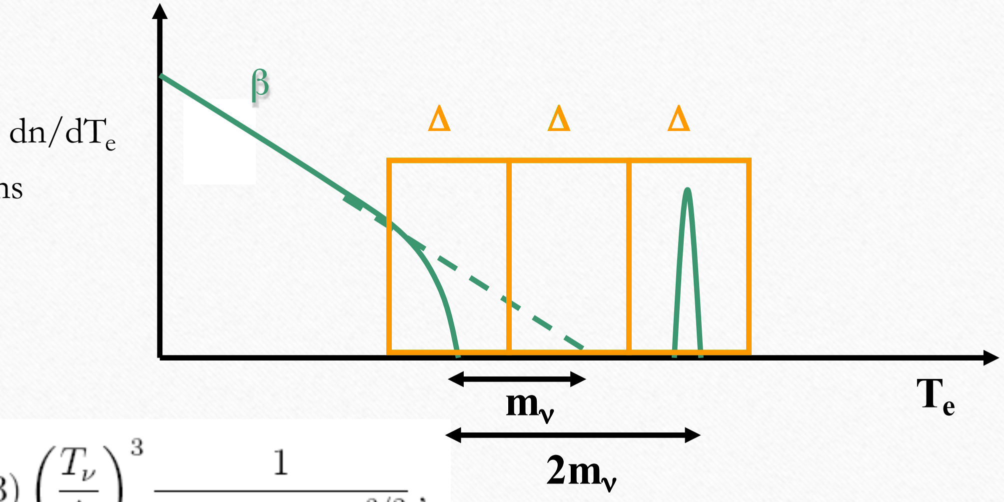


1272 β⁻ nuclei

799 β⁺ nuclei

2. Background

Observing the last energy bins of width **Δ**



$$\frac{\lambda_\nu}{\lambda_\beta(\Delta)} = \frac{9}{2}\zeta(3)\left(\frac{T_\nu}{\Delta}\right)^3 \frac{1}{(1+2m_\nu/\Delta)^{3/2}},$$

signal/background >1

$$\frac{9}{2}\zeta(3)\left(\frac{T_\nu}{\Delta}\right)^3 \frac{1}{(1+2m_\nu/\Delta)^{3/2}\,\rho} \geq 1,$$

$$\rho = \frac{1}{\sqrt{2\pi}}\int_{2m_\nu/\Delta-1/2}^{2m_\nu/\Delta+1/2} e^{-x^2/2}dx.$$

It works for $\Delta < m_\nu$

The case of ^{3}H

$$\lambda_\beta = 2.85 \cdot 10^{-2} \frac{\sigma_{\rm NCB} v_\nu / c}{10^{-45} {\rm cm}^2} \, {\rm yr}^{-1} \, {\rm mol}^{-1} \, . \qquad \sigma_{\rm NCB}(^3{\rm H}) \frac{v_\nu}{c} = (\, 7.84 \pm 0.03 \,) \times 10^{-45} \, {\rm cm}^2 \, ,$$

m_ν (eV)	FD (events yrs^{-1})	NFW (events yrs^{-1})	MW (events yrs^{-1})
0.6	7.5	90	150
0.3	7.5	23	33
0.15	7.5	10	12

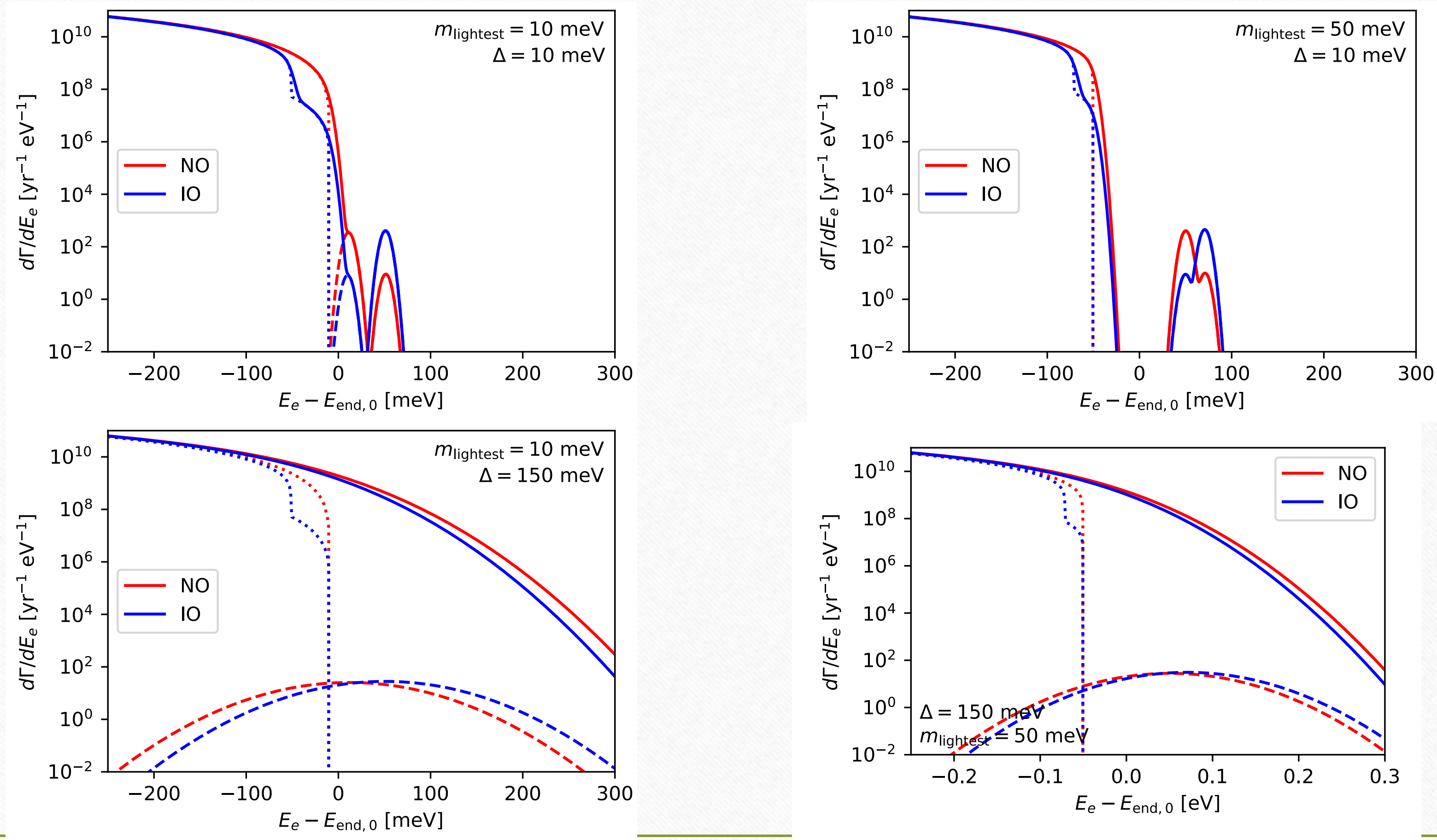
The number of NCB events per year for 100 g of ^{3}H

8 events yr^{-1} per 100g of ^{3}H (no clustering)

up to 10^2 events yr^{-1} per 100 g of ^{3}H due to clustering effect

signal/background = 3 for Δ=0.2 eV if m_ν=0.7 eV

Δ=0.1 eV if m_ν=0.3 eV

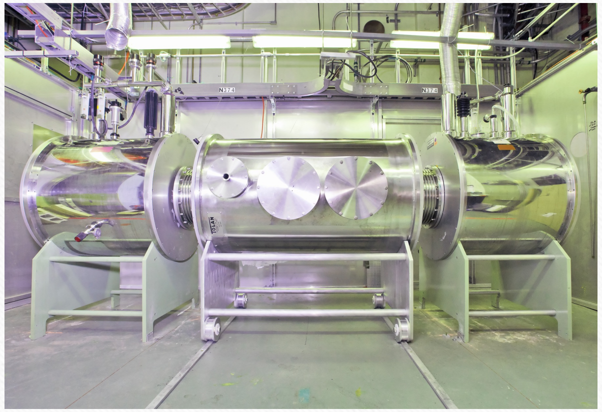


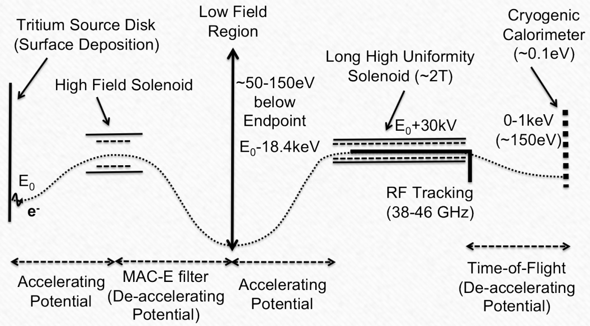
$m_{\mathrm{lightest}} = 10$ meV
$\Delta = 10$ meV
NO
IO
$d\Gamma/dE_e$ [yr^{-1} eV^{-1}]
10^{10} 10^{8} 10^{6} 10^{4} 10^{2} 10^{0} 10^{-2}
−200 −100 0 100 200 300
$E_e - E_{\mathrm{end,0}}$ [meV]
$m_{\mathrm{lightest}} = 50$ meV
$\Delta = 10$ meV
NO
IO
$d\Gamma/dE_e$ [yr^{-1} eV^{-1}]
10^{10} 10^{8} 10^{6} 10^{4} 10^{2} 10^{0} 10^{-2}
−200 −100 0 100 200 300
$E_e - E_{\mathrm{end,0}}$ [meV]
$m_{\mathrm{lightest}} = 10$ meV
$\Delta = 150$ meV
NO
IO
$d\Gamma/dE_e$ [yr^{-1} eV^{-1}]
10^{10} 10^{8} 10^{6} 10^{4} 10^{2} 10^{0} 10^{-2}
−200 −100 0 100 200 300
$E_e - E_{\mathrm{end,0}}$ [meV]
NO
IO
$\Delta = 150$ meV
$m_{\mathrm{lightest}} = 50$ meV
$d\Gamma/dE_e$ [yr^{-1} eV^{-1}]
10^{10} 10^{8} 10^{6} 10^{4} 10^{2} 10^{0} 10^{-2}
−0.2 −0.1 0.0 0.1 0.2 0.3
$E_e - E_{\mathrm{end,0}}$ [eV]

The Ptolemy Project

Development of a Relic Neutrino Detection Experiment at PTOLEMY:
Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield

Pontecorvo





INFN Laboratori Nazionali del Gran Sasso, Italy,

$$\frac{d\widetilde{\Gamma}_{\rm CNB}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi}(\Delta/\sqrt{8\ln 2})}\sum_{i=1}^{N_\nu}\Gamma_i \times \exp\left\{-\frac{[E_e-(E_{\rm end}+m_i+m_{\rm lightest})]^2}{2(\Delta/\sqrt{8\ln 2})^2}\right\},$$

For the fiducial model, the number of expected events per energy bin is given by:

$$\hat{N}^i = N^i_\beta(\hat{E}_{\rm end}, \hat{m}_i, \hat{U}) + N^i_{\rm CNB}(\hat{E}_{\rm end}, \hat{m}_i, \hat{U})\,. \tag{3.3}$$

The total number of events that will be measured in a bin is the sum of $\hat{N}^i$ and a constant background:

$$\hat{N}^i_t = \hat{N}^i + \hat{N}_b\,. \tag{3.4}$$

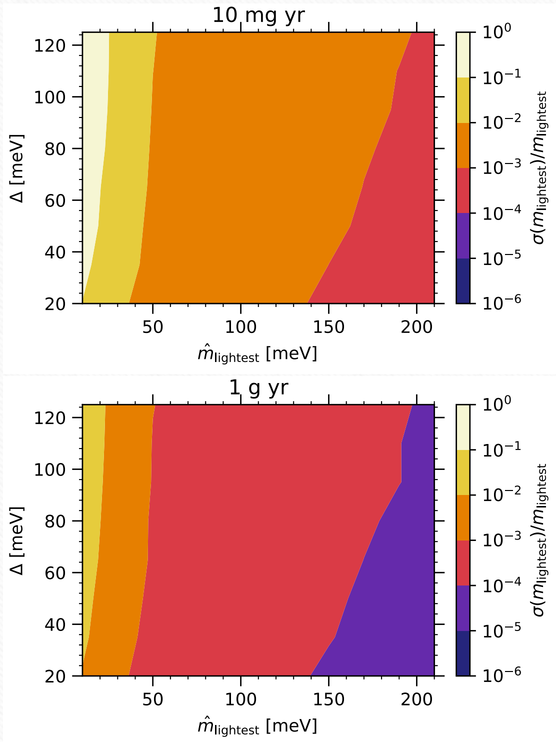
$$N^i_{\rm exp}(\hat{E}_{\rm end}, \hat{m}_i, \hat{U}) = \hat{N}^i_t \pm \sqrt{\hat{N}^i_t}\,,$$

$$\begin{aligned} N^i_{\rm th}(\boldsymbol{\theta}) &= N_b + A_\beta\, N^i_\beta(\hat{E}_{end} + \Delta E_{\rm end}, m_i, U) \\ &+ A_{\rm CNB}\, N^i_{\rm CNB}(\hat{E}_{end} + \Delta E_{\rm end}, m_i, U)\,. \end{aligned} \tag{3.6}$$

In order to perform the analysis and fit the desired parameters $\boldsymbol{\theta}$, we use a Gaussian χ^2 function:

$$\chi^2(\boldsymbol{\theta}) = \sum_i \left(\frac{N^i_{\rm exp}(\hat{E}_{\rm end}, \hat{m}_i, \hat{U}) - N^i_{\rm th}(\boldsymbol{\theta})}{\sqrt{N^i_t}}\right)^2, \tag{3.7}$$

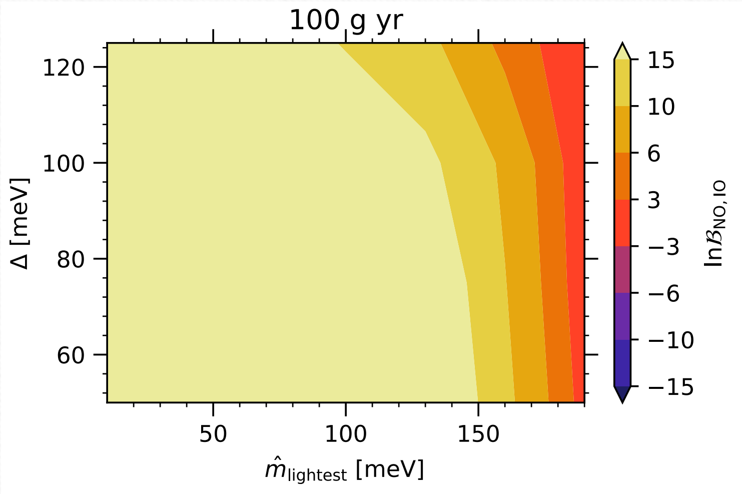
Neutrino mass sensitivity



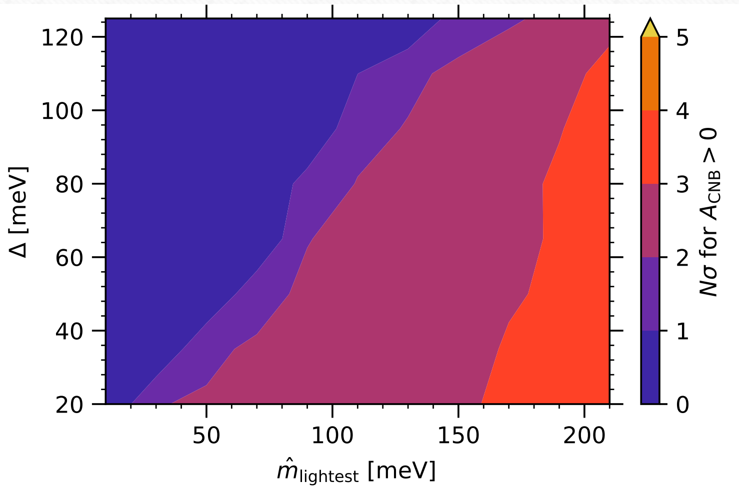
Mass Ordering (Hierarchy)

Bayesian evidence

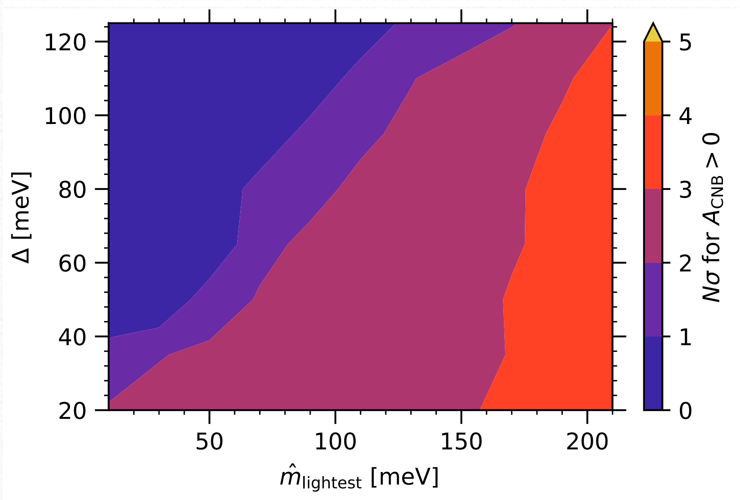
$$\frac{\int d\,\Theta\, P(d|M1)P(M1)}{\int d\,\Theta\, P(d|M2)P(M2)}$$



CNB detection (100 g)



Normal ordering



Inverted ordering

eV sterile neutrinos (100 g)