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Nonlocality, nonseparability and hidden variables in Quantum Mechanics

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Introduction

What is reality? Is there a rigorous way to describe, and therefore predict, the world that surrounds us? Mankind asked itself these question from the very first moment it appeared on the Earth as an intelligent species. Curiosity fed the constant quest for answers. First the field of investigation called «Philosophia Naturalis» was born, and then, with the development of mathematics, it gradually evolved into the natural and physical science the way we know them nowdays. Intuition, simplification and common sense have been for centuries the golden rules to which every theory or model had to subjugate in order to be considered clear and self-consistent. Nevertheless the so-called orthodox interpretation of the leading and most powerful tool to depict reality, Quantum Mechanics, seems to lack at least two of the previously listed features. And yet it works, and it is more efficient than every other theory has ever been before.
In chapter one firstly I will give a definition of good theory and adumbrate a vademecum to follow in order to construct one, secondly I will exhibit why the orthodox interpretation of Quantum Mechanics does not fulfill those requirements. Light will be shed on the problems regarding the observer, measurements, probability and time evolution paying particular attention to the collapse of the wave function.
In chapter two I will illustrate how a reasonable mechanics of the microscopic world should be built and what is the price to pay in order to face phenomena at a scale so different from the classical one. The key concepts of *hidden variables* and *non-locality* will be introduced, also showing that the discardment of a hidden variables theory is unjustified and only caused by a misunderstanding of some proof. In the end a need for *nonseparability*, and perhaps *holism*, will violently emerge from the statements made in this chapter.
In chapter three I will describe David Bohm's interpretation of Quantum Mechanics, display its predictive equivalence to the orthodox one, and point it out as the main candidate and starting theory to develop a new description of reality.
May the reader forgive me if he finds this work too meta-physical.
Let us start!

Chapter 1

Analysis of the Orthodox Interpretation

1.1 Epistemology

Before we may feel ready to analyze and criticize a physical theory, firstly we ought to outline what are the *purposes* and *limits* of physics.
Can we, through science and experimentation, infer a picture of the world? Or should our approach be more pragmatic since we cannot obtain a representation of reality, but only build tools to efficiently operate within it?
Last idea was shared by Niels Bohr who, when asked whether the algorithm of quantum mechanics could be considered as somehow mirroring an underlying quantum world, would answer:

> *''There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.''*[1]

This way of thinking, based on *pragmaticism*, has got the upper hand on the other with the arrival of modern physics. As a matter of fact the father of relativity, Albert Einstein, said:

> *''Physical concepts are free creations of the human mind, and are not, however it may seem, uniquely determined by the external world. In our endeavour to understand reality we are somewhat like a man trying to understand the mechanism of a closed watch. He sees the face and the moving hands, even hears its ticking, but he has no way of opening the case. If he is ingenious he may form some picture of a mechanism which could be responsible for all the things he observes, but he may never be quite sure his picture is the only one which could explain his observations. He will never be able to compare his picture with the real mechanism and he cannot even imagine the possibility or the meaning of such a comparison. But he certainly believes that, as his*

> *knowledge increases, his picture of reality will become simpler and simpler and will explain a wider and wider range of his sensuous impressions. He may also believe in the existence of the ideal limit of knowledge and that it is approached by the human mind. He may call this ideal limit the objective truth.''*[2]

Observation 1. *Therefore the purpose of science is constructing a representation of reality that will asymptotically approach the* objective truth*, but will never reach it. Whatever physical theory we may be able to construct will never be nor* complete *nor* exact*.*

But let us not get demotivated, since even if not complete, a good theory could always be useful.

1.2 A good theory

How do we define a good theory? It comes without saying that it should be *consistent* and, given some initial data, has to provide acceptable[1] previsions in agreement with experimental evidence. In addition it has to respect the well-known falsifiability principle. The latter is somehow built-in in Occam's razor[2], a heuristic technique vastly used both in the development of theorethical models and both as a guide for scientists as sir Isaac Newton [4], Albert Einstein [3], Pierre Maupertuis, Leonhard Euler[5], Max Planck, Werner Heisenberg, Louis de Broglie [6] and many others.

The principle, formulated for the first time by the franciscan friar William of Ockham, was edited and improved by later philosophers and can be resumed as follows[18]:

Observation 2. *Among competing hypotheses, the one with the fewest assumptions should be selected[17].*

If for example we are to apply the razor and choose between two laws A and B describing the behaviour of football players and referees during a football match, with A declaring: «When a defender commits a fault in his penalty area a penalty kick will be assigned to the opposing team.» and B stating: «When a player commits a fault in his penalty area a penalty kick will be assigned to the opposing team, *except if the footballer plays for Juventus F.C.*» we must pick A, the one with less assumptions, and discard B until proved otherwise[3]. It can also come in handy the rule of «epistemological good sense» which Einstein so summarized in a letter to his friend Maurice Solovine:

[1] Within a preset range.

[2] A simpler theory is better testable.

[3] And sadly it has.

- *‘‘The E (experiences) are given to us.*
- *A are the axioms from which we draw consequences. Psychologically the A are based upon the E. There is however no logical path from the E to the A, but only an intuitive (psychological) connection, whic is always ”subject to revocation.”*
- *From A, by a* logical path, *particular assertions S are deduced - deductions which lay claim to being right.*
- *The S are related to the E (testing against experience). Carefully considered this procedure also belongs to the extra-logical (intuitive sphere), because the relations between concepts appearing in S and experiences E are not of a logical nature. This relation of the S to the E is, however (pragmatically), far less uncertain than the relation of the A to the E.’’[9]*

With these rules in mind we can procede to analyze the orthodox theory of quantum mechanics: the Copenhagen interpretation.

1.3 Ontology in the orthodox interpretation

Most students when facing quantum mechanics for the first time are mesmerized by the counterintuitive aftermaths of its postulates. Despite an initial rejection of the theory, the remarkable precision in its previsions convinces the scholar to abandon common sense, or at least to postpone the understanding of the ontological problematics to a second time and concentrate his attention on the numerical results. This approach is perfectly depicted by the quote attributed to Richard Feynman: «Shut up and calculate!»
The objects of the theory, vectors $|\Psi\rangle$ of an abstract separable Hilbert space $\mathscr{H}$, are a *full representation* of the state of a mechanical system at a given time t. The squared amplitude of the projection $c_n = \langle o_n|\Psi\rangle$ of $|\Psi\rangle$ on an eigenstate $|o_n\rangle$ of a Hermitian operator[4] O is the probability of getting the value o_n as the result of the measurement of the observable O on the ket $|\Psi\rangle$. Since the set $\{c_n\}$ forms a complete orthogonal basis for $\mathscr{H}$ we can express the state as

$$|\Psi\rangle = \sum_n \langle o_n|\Psi\rangle\, |o_n\rangle \tag{1.1}$$

We are talking about probabilities, therefore there is a statistical component deeply enrooted in the core of the theory. What the orthodox interpretation is telling us is not that there is a state about which we can not know everything and so we have to manage our knoweledge of it as a statistical ensemble, but rather that the vector, alias the *full* description of the state itself, is a linear combination of the $|o_n\rangle$'s weighted by the probability amplitudes c_n[14]. Werner Heisenberg was amongst the firsts to notice the implications of this definition:

[4]Every physical observable is injectively related to a Hermitian operator. The operation of measurement is the action of the operator on the state-vector. The result will be an eigenvalue[14].

> *''The atoms or elementary particles themselves are not real; they form a world of potentialities or possibilities rather than one of things or facts.''*[7]

Albert Einstein, while acknowledging the incredible benefits in making this assumption, was not so enthusiastic about the direction where the ontology of the theory was heading. He could not bear the idea according to which a thing assumes its *real* status only when measured. Henceforth he preferred not to impose *intrinsically* a statistical trait into the very essence of reality (or what we can tell about it) and suggested a way to improve the theory:

> *''[...] the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics within the framework of classical mechanics. I am rather firmly convinced that the development of theoretical physics will be of this type; but the path will be lenghty and difficult.''*[15]

Observation 3. *Enforcing statistics into the state corresponds to placing a* structural limit *into the theory.*

While on one hand it is true that perhaps that is what we can say about a system, on the other it denies the chance to improve the description, thus neglecting what stated in the first section of this chapter, and putting our pursue of the «objective truth» to an end. Furthermore, in my honest opinion, considering the state as a *superposition* of individually detectable eigenstates does not remove its real soundness but endows it with the chance of being more things at the same time. Ok, the theory works, but if we were to determine an equivalent one that does not require this counterintuitive extra-feature we might be able to reject the orthodox interpretation in accordance with Occam's razor.

1.3.1 Time evolution and collapse

As already mentioned, the evolution of a ket $|\psi\rangle(t_0)$ at a given time t_0 is dictated by two laws: 1) Schrödinger equation, 2) wave function[5] collapse[14].

1. Schrödinger equation is a *deterministic* differential dynamical law:

$$i\hbar\frac{d|\psi\rangle}{dt} = H|\psi\rangle \tag{1.2}$$

   with H being the Hamiltonian operator for the system. By Cauchy's theorem it has a unique solution:

$$|\psi\rangle(t) = e^{\frac{-i}{\hbar}H(t-t_0)}|\psi\rangle(t_0) \tag{1.3}$$

[5] The representation of the abstract vector in the position basis. So-called because the first formulation of quantum mechanics was derived in an attempt to construct a sort of wave mechanics.

As a consequence known a state at a given time, the state at any time t is fixed by the action upon it of a unitary operator $e^{\frac{-i}{\hbar}H(t-t_0)}$.

2. Wave function collapse is a *probabilistic* law of evolution.
When an observer measures an observable O of the system (expressed as (1.1), the state-vector *instantaneously* collapses into one of the eigenstates $|o_n\rangle$ with probability $|c_n|^2$.

The introduction of the law of collapse, and the momentary abrogation of Schrödinger equation, seems a gimmick to put the dust under the carpet and do not accept that perhaps there is something more, that could be known, which the theory only partially describes.

While talking of unanimated objects a superposition of states can be digested, this idea becomes even more absurd when transposed to living beings. The famous paradox of the Schrödinger cat is perhaps the most noteworthy example of lack of classical logic intuition[14]. The cat is in a state that is a linear combination of Alive and Dead. What does it mean? I, who am outside the box in where it is bounded, do not know if it is alive or dead, but does not it measure itself? Does it not perceive its own changes?

Observation 4. *As a matter of fact, for it to work properly, the collapse needs a yet vague separation of the world in two categories: the observers with their measurement devices and the system to be measured.*

1.3.2 Wave-particle duality

Although our sensibile experience is made of interactions with localized[6] particles, a system is described by a wave function which is defined (almost) *everywhere*. The latter evolves according to Schrödinger equation. As believed by the orthodox theory this means that if we try to localize a particle there is a chance different than naught to find it in very different and distant places at short time intervals. Does the particle violate the *locality principle*? Perhaps yes, as we will see in the next chapter. Thus, how can the classical laws of kinematics hold for a macroscopic body? Ehrenfest theorem[14] states that they are preserved not for the position of a particle, but for its expectation value. Hence the shape of the modulus squared of a wave function for a finely localized particle should be one resembling a *packet* with amplitude decreasing the more it drifts away from its average value.[7] The problem lies in the fact that Schrödinger equation prescribes a spread of any bundle of wave functions with the passing of time (Figure 1.1). In the end in the universe there won't be any pinpointed body.

What saves the orthodox theory from this incompatibility with reality is the law of collapse.

[6]With localization in phase-space being limited by Heisenberg's uncertainty principle.

[7]Usually gaussian distributions serve the purpose.

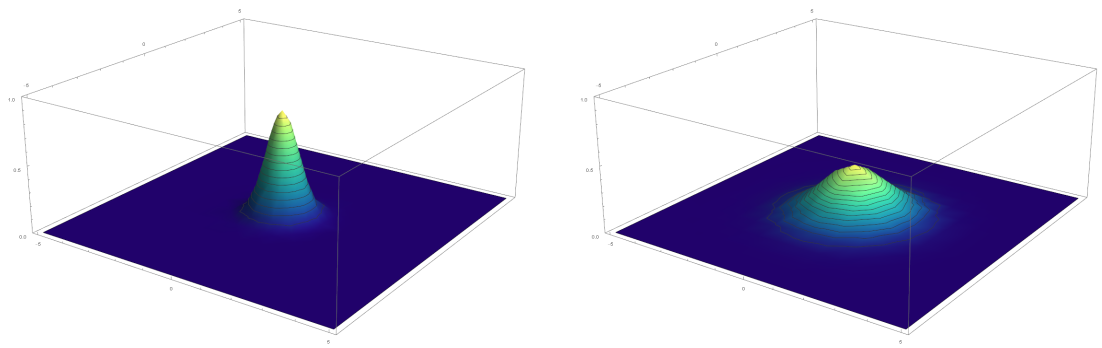

Figure 1.1. A 2D wave packet $|\psi(\vec{r}, t)|^2$ at time t^* and later at $t^* + \Delta t$

Observation 5. *When not measured a previously localized particle behaves like a wave, it spreads, but when measured it instantenously get localized again.*

It's such a muddled explanation!

David Bohm proposed a different interpretation of quantum mechanics by which an object is composed of a particle *and* a wave simultaneously, with both always mantaining their structure independently of measurement[13]. The particle will respect time-dependent law of motion. This will be the topic of Chapter 3.

1.3.3 On the measurer and measurement

A more fundamental issue arises from (Observation 4) and John Bell promptly criticized it:

> *''[...] It would seem that the theory is exclusively concerned with 'results of measurement' and has nothing to say about anything else. When the 'system' in question is the whole world where is the 'measurer' to be found? Inside, rather than outside, presumably. What exactly qualifies some subsystems to play this role? Was the world wave function waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some more highly qualified measurer - with a Ph.D.? If the theory is to apply to anything but idealized laboratory operations, are we not obliged to admit that more or less 'measurement-like' processes are going on more or less all the time more or less everywhere? Is there ever then a moment when there is no jumping and the Schrodinger equation applies? The concept of 'measurement' becomes so fuzzy on reflection that it is quite surprising to have it appearing in physical theory at the most fundamental level. [...] And does not any analysis of measurement require concepts more fundamental than measurement? And should not the fundamental theory be about these more fundamental concepts?''*[12]

Or quoting Richard Feynman's words:

> *''This is all very confusing, especially when we consider that even though we may consistently consider ourselves to be the outside observer when we look at the rest of the world, the rest of the world is at the same time observing us, and that often we agree on what we see in each other. Does this then mean that my observations become real only when I observe an observer observing something as it happens? This is a horrible viewpoint. Do you seriously entertain the idea that without the observer there is no reality? Which observer? Any observer? Is a fly an observer? Is a star an observer? Was there no reality in the universe before* 10^9 *B.C. when life began? Or are you the observer? Then there is no reality to the world after you are dead? I know a number of otherwise respectable physicists who have bought life insurance.''*[10]

Theorists claim for themselves the right to decide what is part of the system and what is outside it. *FAPP*[8] this assumption could be useful, nevertheless it starts to carve a deep crack into the soundness of the theory. I share the same perplexities as Bell's and Feynman's. It is preposterous to conceive that the most accurate description of objects at their most elemental level is completely determined in the act of measurement by some out-of-the-box apparatus, whose action is arbitrary fixed by an again out-of-the-box human being. Although we had to accept a statistical ontology of reality, at least we would require a more strictly regulated law of time evolution which would not be so unclear. It is quite obvious that the measurement device and the measurer are macroscopic agents that are considered as a true[9] statistical average of quantum properties. We already stated that the line between the macroscopic and the quantum world is at the own discretion of the physicists. Features "above" this line are averages of the ones below it. We can imagine to push the line into the manifest quantum world and beyond it.
Therefore quantum characteristics themselves are to be considered averages of some other, more fundamental, hidden variables.

Observation 6. *To this extent the wave function would only provide the probabilities of every possible microscopical configuration.*

Why then has not a hidden variable formulation become the standard interpretation? We have to blame a misunderstanding of the conclusions of several theorems by Von Neumann, Bell[12], and others which made people affirm that mere ignorance cannot be accounted for the fluctuations and the bizzarre events of the quantum world[8].
As we will see what the theorems truly say is subtle but deeply different.

[8]For all practical purposes.

[9]In the classical sense.

Chapter 2

Non-locality and Hidden Variables

The *principle of locality* states that an item is directly influenced only by its immediate sorroundings[17]. A theory is *local* if it is endowed with this property. The dimension of the region in space-time sorrounding an object, which can influence or be influenced by it, is fixed by the laws of Special Relativity[1][11]. Any sort of information cannot propagate in space with a speed faster than light. Quantum Mechanics (based on the Copenhagen interpretation) and Special Relativity are unified in a larger theory called *QFT* or *Quantum Field Theory*.

In the last part of the previous chapter we came to the conclusion that a hidden variables formalism of the quantum theory would provide a more reasonable, intuitive and *complete* recipe for the description of the physical world.

Let $\boldsymbol{B}$ being the variables of a quantum theory. The incompleteness thesis for the theory states that whatever mathematical representation of the quantum formalism in terms of variables $\boldsymbol{B}$, and whatever criterion used to associate the initial condition of a given state of macroscopical objects to the variables $\boldsymbol{B}$ which represent it, the final condition of the $\boldsymbol{B}$ variables does not corresponds to the state of objects at the final time[8]. A hidden variable theory introduces a set of variables $\boldsymbol{A}$ which provide a more complete description of a state in every situation and are independent of any act of measurement. Orthodox theory rejects this approach and yet it admits in a subtle way that the only variables $\boldsymbol{B}$ are not enought to completely depict a state. Sure enough, Bohr's interpretation fills the gap left by the $\boldsymbol{B}$'s including another set of variables $\boldsymbol{C}$ which is the set of *classical variables*. As concerns this, Landau and Lifshitz said:

> *''A more general theory can usually be formulated in a logically complete manner, independently of a less general theory which forms a limiting case of it. [...] It is in principle impossible, however, to formulate the basic concepts of quantum mechanics[2] without using classical mechanics.''*[16]

[1] As of today no theory is able to consistently unite General Relativity and quantum mechanics.

[2] *As for the orthodox interpretation*

So what we have to do is to replace a model based on the set of variables (B, C) with one which includes also (or better only, if possible) the A's: (A, B, C).

2.1 Refusal of Von Neumann's proof

All of this aside John von Neumann in 1932 presented an argument which has been for decades the tombstone of any hidden variables intepretation[19]. His first assumption was: «Any real linear combination of any two Hermitian operators represents an observable, and the same linear combination of expectation values is the expectation value of the combination».
This is valid for quantum mechanical states described by the (B, C). If a state described by the A's existed it would be not depicted by a probability amplitude but its properties would be perfectly localized with no dispersion. Von Neumann firstly speculated ab absurdum the existence of these dispersion-free states and secondly demonstrated straightforwardly from the previously written assumption that the existence of such states contradicted it. We are not to forget that the mathematical framework of a theory is only employed for modeling a real, physical process like a happening, an experiment or a measurement. While it is easy to define an operator as the sum of two non-commuting ones, it is not straightforward to actually find an experiment that measures an observable related to that specific operator. We say that every observable is injectively related to a Hermitian operator thus the opposite is not given for granted. If we intend for the new observable we are going to averagely measure the trivial combination of the averages of the single well-defined observables, then it is well known that we cannot consider the simple sum of the data gained by the separate experiments because of the non-commutativity of the operators. We need a totally different experiment that, as stated before, could not be realized. As Bell phrased:

> *''But this explanation of the nonadditivity of allowed values also established the nontriviality of the additivity of expectation values. The latter is a quite peculiar property of quantum mechanical states, not to be expected a priori. There is no reason to demand it individually of the hypothetical dispersion free states, whose function it is to reproduce the measurable peculiarities of quantum mechanics when averaged over.''*[12]

In light of the considerations that Bell (and us) have shown up to this point we can declare that Von Neumann's proof did not suffice to rule out a reinterpretation of quantum mechanics in terms of hidden variables.

2.2 Bell's Theorem

After a deep examination of David Bohm's theory which was manifestly non-local, Bell realized that perhaps locality was not a necessary feature:

> *''[...] there is no proof that any hidden variable account of quantum mechanics must have this extraordinary character.''*[12]

After he had analyzed the *EPR*[3] paradox Bell gained an inequality that regulated the relationship between a hidden variable formalism and locality. Later J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt improved the argument and obtained a more general statement.

2.2.1 The EPR paradox

In 1935 EPR published an article with the intention of establishing the incompleteness of the orthodox theory and consequently expanding it to a more general one by inlcuding some hidden variables in its formulation. As we have already adumbrated in Chapter 1, the intrinsic statistic property of quantum mechanics does not grant a system with a unique, pre-determinate, real status. The state-vector gains it, and only for a short time, with the process of collapse after a measurement. The wave function for a many particle system can be a linear arrangement of the direct product of the wave function for the single particles. For indistinguishable particles the combination has to be symmetric under permutations, therefore the complessive state has to be totally symmetric or anti-symmetric. [20]
Let us create a couple of spin 1/2 *entangled* particles in the so-called singlet state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle) \tag{2.1}$$

with $\{|\uparrow\rangle, |\downarrow\rangle\}$ being an orthonormal basis for the spin space. Let the particles now travel away from each other in a way such that their spin anti-correlation is not destroyed. When we measure the spin of a particle in a chosen direction the total state $|\psi\rangle$ collapses unto the state $|\uparrow\rangle \otimes |\downarrow\rangle$ or unto $|\downarrow\rangle \otimes |\uparrow\rangle$. Hence measuring the spin of the other particle in the same direction we will have 100% chance of finding it pointing to the other side. Therefore the interaction of the first particle with the measurement apparatus instantenously determined the state of the second particle which can be very distant in space from the first one. This is a violation of locality. We can reiterate the procedure for every possible direction and conclude that:

Observation 7. *The orthodox theory cannot provide a complete description of reality since it assumes locality. To preserve the latter the particles should have a pre-existing spin in every possible direction. This contradicts the assumption that the state vector is not a mere statistical description of reality.*

[3] Einstein-Podolski-Rosen

2.2.2 Bell-CSCH inequalities

I will now illustrate the inequalities derived by Bell following his proof step by step[12]. Let us consider the state (2.1).
We label by a variable $A = +1$ or -1 the output of the measurement respectively of $|\uparrow\rangle$ or $|\downarrow\rangle$ for the first particle, and $B = +1$ or -1 for the other. We denote a and b the angles by which we have rotated our Stern-Gerlach magnets[4] to measure the spin of the two particles from a previous direction they were aligned along. Let c be a set of variables describing the two experimental devices in a region of the backward lightcones for particle 1 and 2 where they no longer overlap (Figure 2.1). Let also λ be a set of additional variables to complete the description of the system in the way prescribed by EPR.

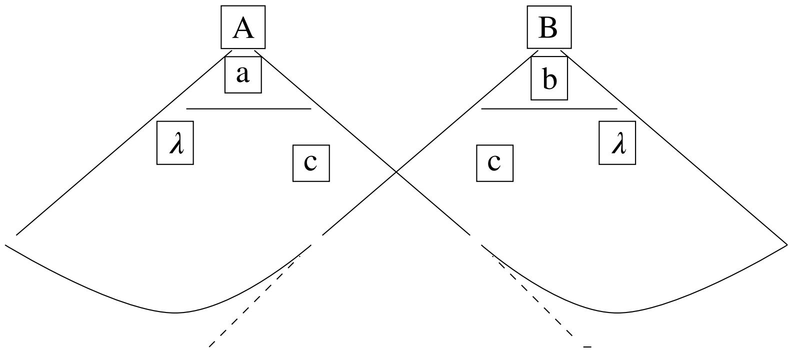

Figure 2.1. Space-time distribution of variables

Then

$$\{A, B|a, b, c, \lambda\} \tag{2.2}$$

represents the probabilities of observing particular values of A and B given a specific configuration of the variables listed in the right side of the bracket. Therefore the joint probability is

$$\{A, B|a, b, c, \lambda) = \{A|B, a, b, c, \lambda\}\,\{B|a, b, c, \lambda\} \tag{2.3}$$

Let the a's and the b's be free variables: totally independent of the c's and the λ's. It is an exagerrated assert, since it means that the people who are performing the measurement can pick a *random* orientation for the magnets, but it will serve the purpose as the limiting case of more physical acceptable correlations.
Requesting local causality and then invoking the assumed completeness of the variables c and λ in the region of space-time where they are defined, we can suppress the explicit

[4]That we use in order to observe the spin. In the original article by Bell the author referred to polarizers rather than magnets.

dependence of the right side in the brackets in (2.3) from variables with a space-like separation.

$$\{A, B|a, b, c, \lambda) = \{A|a, c, \lambda\}\,\{B|b, c, \lambda\} \tag{2.4}$$

Equation (2.4) tells us that the value of A cannot depend on B and vice versa. More specifically A and B depend only on the local configurations of the magnets a and b, on the hidden variables λ and on the standard configuration of c in the already non-overlapping space-time region. Let us now assume that the rotation of the magnets is totally free and not influenced anyhow by a previous configuration of the environment, in this way the λ's will depend only on the c. Then we define a correlation function E as the expectation value of the product of A and B [12]

$$E = \sum_{\lambda} \sum_{A,B} AB\,\{A, B|a, b, c, \lambda\}\,\{\lambda|c\} \tag{2.5}$$

Therefore, invoking (2.4), for every configuration (a, b, c) we have:

$$E(a, b, c) = \sum_{\lambda} \sum_{A,B} AB\,\{A|a, c, \lambda\}\,\{B|b, c, \lambda\}\,\{\lambda|c\} \tag{2.6}$$

It is straightforward[5] to obtain the following inequality from (2.6):

$$|E(a, b, c) - E(a, b', c)| + |E(a', b, c) + E(a', b', c)| < 2 \tag{2.7}$$

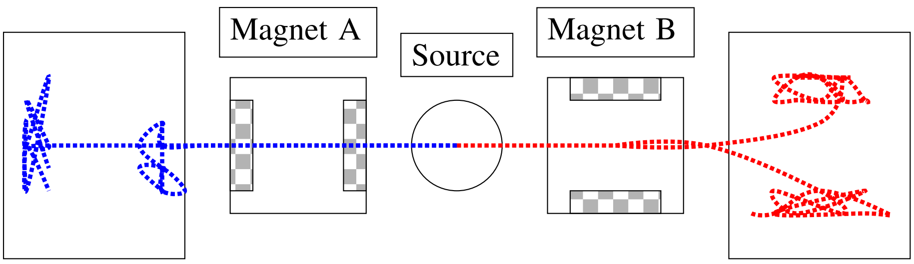

Figure 2.2. Stern-Gerlach experiment

The correlation (2.6) for a quantum system[6] with the c's already included in the preparation of the experiment (Figure 2.2) can be expressed as:

$$E(a, b) = \langle \psi|\sigma_a^1 \sigma_b^2|\psi\rangle = -\cos(a - b) \tag{2.8}$$

[5]Directly from probability theory.

[6]In the orthodox theory which does not predict the existence of the λ's.

with σ_n^i the operator which measures the spin of the i particle along the z axis rotated by n. If for example $a = \frac{\pi}{4}$, $b = \frac{\pi}{2}$, $a' = 0$ and $b' = \pi$ then (2.7) becomes

$$|E\left(\frac{\pi}{4},\frac{\pi}{2}\right) - E\left(\frac{\pi}{4},\pi\right)| + |E\left(0,\frac{\pi}{2}\right) + E(0,\pi)| = 1 + \sqrt{2} \nleq 2 \tag{2.9}$$

which is a clear violation of the original (2.7).

Bell's Theorem. *«In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant.»[12]*

2.3 Entanglement and faster-than-light signaling

The conclusions of the last section are quite remarkable: an inner nonlocal nature emerges from some peculiar quantum configurations. In particular it's beyond a shadow of doubt that two spin-entangled particles can, in principle, anyhow interact with each other even though their separation is space-like. Nevertheless it has been proved in the famous No-communication theorem, by assuming the separability of the total Hilbert space wherein the kets live, that although some sort of nonlocality seems crawl around in the quantum world, it is impossible to exploit it in order to transfer information faster than the speed of light[21].

Antony Valentini suggested in 1991 that there is a *fine-tuning problem* in Quantum Mechanics and that «Quantum theory is a special case of a wider physics.»[22][23] The explanation is strictly linked to the concept of *quantum equilibrium* that we will adumbrate in detail in the next chapter. To make a long story short, Valentini claims that the postulates of the Copenhagen interpretation, according to which the squared modulus of the wave function[7] $|\psi(x)|^2$ is equal to the probability density of finding the point particle in position x, in reality reached this status only after the system had evolved through a process of *relaxation* to a *quantum equilibrium*. In a hidden variable theory only the variables that are in equilibrium correspond to a configuration which would give predictions equivalent to those of the orthodox theory. In the end of the day we could, if proved correct, use non-equilibrium configurations to have faster-than-light signaling[24]. Until that day this is only speculation.

[7] In configuration space.

2.4 Holism and nonseparability

During the derivation of Bell's theorem at a certain point we assumed the total freedom for a and b as the limiting case of a partial, unknown, correlation between (a, b) and c. A drop of complete indeterminism was therefore injected into the theory. What if this freedom were an unnecessary trait? The theory would then be *superdeterministic*.

In a 1985 radio interview to BBC John Bell declared:

> *''There is a way to escape the inference of superluminal speeds and spooky action at a distance. But it involves absolute determinism in the universe, the complete absence of free will. Suppose the world is super-deterministic, with not just inanimate nature running on behind-the-scenes clockwork, but with our behavior, including our belief that we are free to choose to do one experiment rather than another, absolutely predetermined, including the "decision" by the experimenter to carry out one set of measurements rather than another, the difficulty disappears. There is no need for a faster than light signal to tell particle A what measurement has been carried out on particle B, because the universe, including particle A, already "knows" what that measurement, and its outcome, will be.''*

What Bell had in mind, when he spoke regarding a particle which already knows the outcome of an experiment carried out very far away from it, was a reality that lays on the pillar concept of *nonseparability*. In Quantum Mechanics nonseparability[8] implies that we cannot describe the state of a system $|\Psi\rangle$ with the direct product of what we think are its discrete constituents $|\psi_i\rangle$:

$$|\Psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \otimes ... \tag{2.10}$$

Or, more generally, the *separability principle* was defined by Howard in the following way: «The contents of any two regions of space-time separated by a nonvanishing spatiotemporal interval constitute separable physical systems, in the sense that (1) each possesses its own, distinct physical state, and (2) the joint state of the two systems is wholly determined by these separate states.»[25]

Howard made the case that an underlying *ontological holism*[9] is responsible for the violation of the latter. Ontological holism is the main postulate upon which David Bohm built his interpretation of quantum mechanics[13], but in the formulation of the theory his results are partially in contrast with the previous assumption. Bohm's theory is deterministic, and this can not be a direct consequence of holism. Freedom could emerge from a holistic theory. Even though ontological holism entails nonseparability the opposite is not necessarily true[25].

[8] As defined by the following principle by Howard.

[9] Some objects are not entirely made of basic physical parts.

Observation 8. *There is no methodological way to predetermine what kind of properties may emerge from the assumption of holism. Let us immagine a universe that is separable into discrete parts until a certain scale and then it becomes nonseparable. Let us consider one of these parts as a system. If we assume this system to be holistic then it is also nonseparable because it really* *is* *only one thing. If we are able to identify within it some smaller constituents, we will not be able to fully cover all the properties of the system considering the union of the sets of the characteristics of these smaller constituents. The system will have other intrinsic traits that either could or also* could not *depend on the relations between the smaller parts. The latter means that there is a sort of freedom or unforeseeability in what is a complete list of characteristics proper of the system. Whereas nonseparability states that the extra-properties of the system depend only on the relations that the smaller constituents have between themselves, even though the single parts do not have them if taken independently.*

Observation 9. *In order to abide by the epistemology rules listed in Chapter 1 we cannot assign to the universe a non explicitly requested property of holism. Our discussion so far ascertained only nonseparability. Therefore I don't feel like stating that David Bohm's interpretation is correct without further evidence. I rather think that Bohm did not have in mind a clear distinction between the two concepts and he summoned holism instead of invoking nonseparability. Nevertheless, since holism includes and provides an explanation for nonseparability, and in addition to that its picture is also endowed with hidden variables (which as we saw are to be preferred over an intrinsic stastical interpretation), it is by and large a more accurate model than the orthodox theory.*

2.5 The Aharonov-Bohm effect

Yet another experimental evidence of the non-local nature of quantum mechanics is provided by the Aharonov-Bohm effect. In 1959 Aharnonov and Bohm theorized that the interference pattern originated from a ray of charged particles in a double slit experiment could be modified by a constant magnetic field produced by an ideal solenoid, even though that field was confined to a region from which the particles were not allowed[26].
Let us imagine a charged particle passing through two slits separated by d, with l being the distance between the opens and a detector screen, and y the distance between any point on the screen and the middle of it. If right in the center between the slits, just next to the wall, there is a small infinite ideal solenoid whose wide side is orthogonal to the paper (Figure 2.3) there will be a magnetic field $\vec{B}$ confined to the inside of the solenoid, parallel to the $\vec{z}$ direction. In classical physics we could use the *unphysical* vector potential $\vec{A}$ to easen the calculations since $\vec{B} = \nabla \times \vec{A}$. Therefore $\vec{A}$ is defined even out of the solenoid. Being λ the De-Broglie wavelenght of the charged particles, q the electric charge and Φ the magnetic flux through the solenoid, we are going to obtain the interference pattern generated by the split beam when it incides upon the screen.

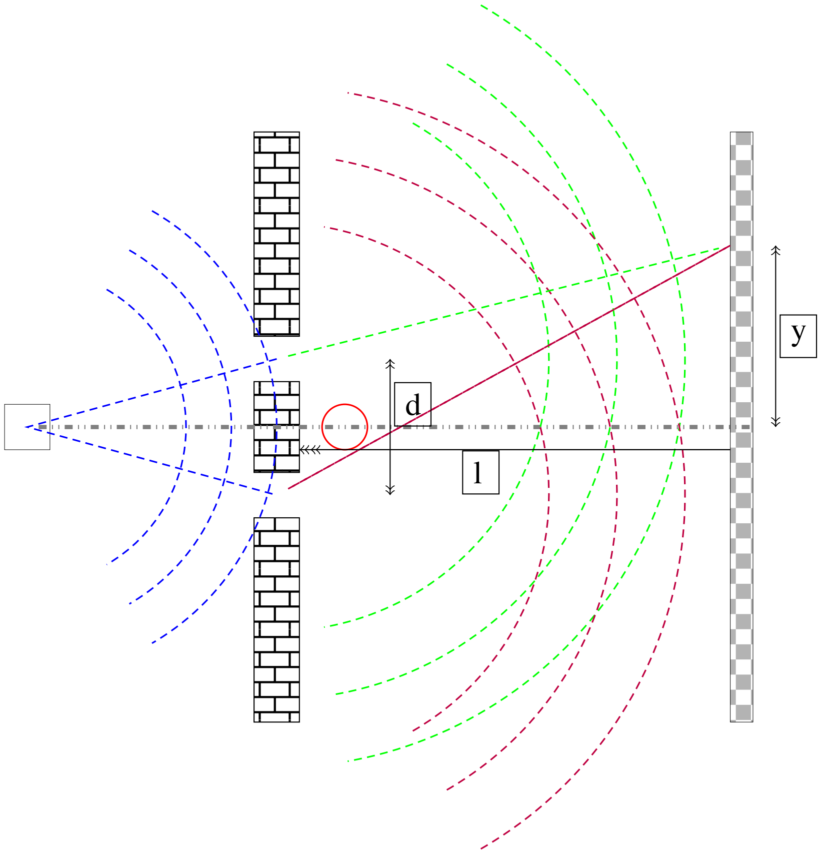

Figure 2.3. Setup for the Aharonov-Bohm double slit experiment

When no current circulates through the solenoid the phase difference δ between the wave which passes through the first slit and the wave which passes through the second is proportional to the difference in path lengths u. For y smaller than l $u \approx \frac{yd}{l}$ and hence $\delta \approx \frac{2\pi yd}{\lambda l}$.

When there is a magnetic flux through the solenoid the wave functions of the charged particles get coupled with the vector potential $\vec{A}$. The latter generates an additional phase difference over a single path:

$$\delta = -\frac{q}{\hbar}\int_{\gamma} \vec{A} \cdot d\vec{r} \tag{2.11}$$

The total phase difference between the two paths will then be:

$$\Delta\delta = \delta_1 - \delta_2 = -\frac{q}{\hbar}\left(\int_{\gamma_1} \vec{A} \cdot d\vec{r} - \int_{\gamma_2} \vec{A} \cdot d\vec{r}\right) = -\frac{q}{\hbar}\oint_{\Gamma} \vec{A} \cdot d\vec{r} \tag{2.12}$$

The integral is to be evaluated over the closed loop Γ which starts from the source, passes through the first slit, goes to the screen and then returns to the source passing through the second slit (circling the solenoid). If $\vec{B}$ is constant (2.11) can be rewritten as:

$$\Delta\delta = -\frac{q}{\hbar}\oint_{\Gamma}\vec{A}\cdot d\vec{r} = -\frac{q}{\hbar}\int_{\Sigma}\left(\nabla\times\vec{A}\right)\cdot d\vec{\sigma} = \frac{q}{\hbar}\Phi \tag{2.13}$$

Where Σ is the spatial region enclosed by the curve Γ. (2.12) is independent of y, therefore the whole interference pattern will be shifted by the same amount proportional to Φ:

$$\Delta y = \frac{l\lambda}{2\pi d}\Delta\delta = \frac{l\lambda}{2\pi d}\frac{q}{\hbar}\Phi \tag{2.14}$$

How can the charged particles be affected by a magnetic field that is nonzero only in a region from where they are excluded? Healey proposes two different explanations:[27]

1. Electromagnetism is *non local*. The magnetic and the electric fields $\vec{B}$ and $\vec{E}$ are physical. The charged particles are affected by the magnetic field due to a non local interaction.

2. Electromagnetism is *local* but *non spacial separable*. The vector and scalar potential $\vec{A}$ and V are physical and more fundamental[10]. We can not effectively separate the potentials assigning to them a spatial *pointwise* value and therefore effect. Every single non self-intersecting loop and the region enclosed by it is allocated with different intrinsic electromagnetic properties.

The first explanation seems a little absurd since Einstein derived special relativity extending electromagnetism itself. Nevertheless, in both cases Bell's inequalities are violated.

The Aharonov-Bohm effect was experimentally confirmed by Chambers in 1960, only one year after its theorization[28].

[10] With a particular gauge choice.

Chapter 3

De Broglie-Bohm theory

In 1927 Louis de Broglie developed a different interpretation of the quantum theory. In 1952 David Bohm independently worked out and then tweaked it. All of the characteristics automatically descend from its dynamical equations that will be adumbrated in the next section. I will resume them, giving emphasis to the main differences between the Bohmian and the orthodox theory:

- A system of physical particle is completely described by their position *and* a wave. In this way a particle has a *definite* position in every instant which plays the role of what would have been a hidden variable for Copenhagen interpretation.

- The wave is a solution of the Schrödinger equation.

- The positions evolve according with a velocity field which is proportional to the gradient of the phase of the wave in a multi-dimensional configuration space.

To some degree it may seem controversial that the Bohmian theory *requires* the orthodox one[1]. More accurately what truly makes the difference is the meaning we give to this wave which drives the world. While in the orthodox representation $|\psi(\vec{q})|^2$ is the probability density of measuring the system in a particular volume element $d\vec{q}$ in configuration space, in the de Broglie-Bohm theory $|\psi(\vec{q})|^2$ is the probability for the system to actually *be* in that volume element $d\vec{q}$. This is a crucial point. The wave function therefore depends on the configuration of the *whole universe*. Moreover, since as we have seen the wave function is strictly related to the velocity field, the latter in turn is non local. Therefore a particle could be affected by changes in the configuration of the universe even though the separation from that given event is space-like.

[1] It requires the Schrödinger equation.

3.1 Dynamics and guidance equation

Let us start to introduce the fundamental equations. Let Σ be a collection of point particles. Following Bohm's prescription[13] the state is completely determined by $(\vec{q}, \psi)$ where $\vec{q}$ is a position in configuration space and $\psi(\vec{q}, t)$ is the wave function associated to the system. ψ evolves according with the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla_k^2 \psi + V\psi \tag{3.1}$$

with V being a scalar potential energy field.

If we write the wave function in exponential form $\psi = Re^{\frac{iS}{\hbar}}$ and define $\rho = R^2 = \psi^* \psi$ we obtain:

$$\frac{\partial S}{\partial t} + \sum_{k=1}^{N} \frac{(\nabla_k S)^2}{2m_k} + V + Q = 0 \tag{3.2}$$

where Q acts as a *quantum potential*

$$Q = -\frac{\hbar^2}{R} \sum_{k=1}^{N} \frac{\nabla_k^2 R}{2m_k} \tag{3.3}$$

and

$$\frac{\partial \rho}{\partial t} + \sum_{k=1}^{N} \nabla_k \cdot (\rho \nabla_k \frac{S}{m_k}) = 0 \tag{3.4}$$

(3.4) is a continuity equation for the quantity ρ and (3.2) is a sort of Hamilton-Jacobi equation with the momenta of the particles being

$$\vec{p}_k = \nabla_k S \tag{3.5}$$

Or, written as a velocity field and returning to the normal expression of the wave function

$$\frac{d\vec{q}_k}{dt} = \frac{\hbar}{m_k} \Im \left[\frac{\psi^* \nabla_k \psi}{\rho} (\vec{q}_1, ..., \vec{q}_N) \right] \tag{3.6}$$

3.1.1 The hydrogen atom

Even though Bohmian Mechanics provides trajectories for quantum phenomena it is not a classical theory; for example in (Figure 3.1) we can see a speculative representation of the possible paths for a particle in a double slit experiment. A striking example is the non-relativistic hydrogen atom. The energy eigenfunctions of the hydrogen atoms in spherical coordinates are

$$\psi_{nlm}(r, \theta, \phi) = R_{nl}(r) Y_l^m(\theta, \phi)$$

where R_{nl} is a real function of the radial part which depends only on the principal quantum number n and the orbital angular momentum l, and Y_l^m is a spherical harmonic. Since only Y_l^m with $m \neq 0$ contribute to the phase S of ψ, (3.5) implies that all the electrons in a state with $m = 0$ are still, while the others revolve in a plane perpendicular to the z-axis.

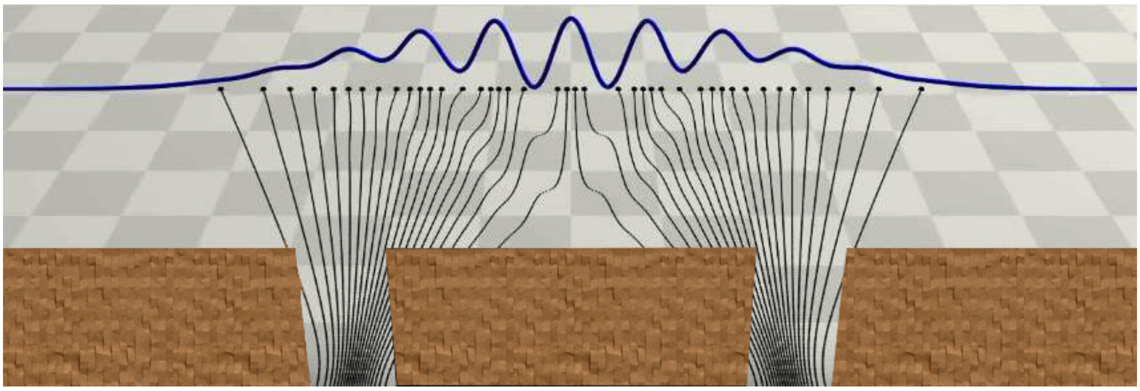

Figure 3.1. Speculative 3D representation of trajectories for particles in a double slit experiment. The blue continuous line represents the final interference pattern.

3.1.2 Guidance equation for particles with spin

How can an electron of a hydrogen atom be still if affected only by the Coulomb potential? Because the action of the latter is perfectly balanced by the Quantum potential! However, we can quell the uprising powered up by our intuition if we take account in the laws of dynamics that the particle also has spin. Coljin and Vrscay take a cue from Holland's work and show that (3.5) should be modified with the addition of an extra-term which takes care of the spin dependence[29]. This term is the Dirac-current or its non relativistic limit: the Pauli current

$$\vec{p}_k = \nabla_k S + \log \nabla \rho \times \vec{s}_k \tag{3.7}$$

where $\vec{s}$ is the spin.

3.1.3 Dynamics for relativistic field theory

In order to extend Bohm theory to one which is valid for relativistic regimens we shall follow the usual QM approach and switch our focus to fields and field equations. Every «observable» in the field theory must have a well defined position in space-time that will play the role of a hidden variable as it did in the previous section.

While the extension for scalar fields is straightforward, the widening of the formalism for the sake of Dirac fields could result a bit tricky. I will follow Bell's presentation of the argument[12].

First of all, if we have to define a position for objects in a full relativistic regimen the standard approach should be to consider the energy-momentum tensor $T_{\mu\nu}(r)$. Its *00* component, the so called energy density, do not commute for different r.
$[T_{00}(r), T_{00}(r')] \neq 0$. Therefore it would be impossible to simultaneously measure energy densities in different positions of space-time. To overcome this hindrance Bell defines a

fermion number density over a latticed 3D space (keeping the time continuous and real). This fermion density will include the positions of every fermion in the universe and extra information. Let i be an index that goes over natural numbers which lists all the points of the lattice

$$i = 1, 2, ..., L$$

where L is very big. We are assuming that the world, once replaced with a very dense lattice, has a finite numbers of points. This is an approximation made in order to simplify the notation and the calculations and is not justified by the presence of any evidence or theoretical argumentations. The lattice point fermion number operators are

$$\Psi(i) = \sum_{\alpha=1}^{N} \sum_{\gamma=0}^{3} \psi^{\dagger}_{\alpha,\gamma}(i)\psi_{\alpha,\gamma}(i) \tag{3.8}$$

where γ's are Dirac indices, α is an index which numbers the N Dirac fields and $\psi_{\alpha}(i)$ and its adjoint are operators that respectively create and annihilate a fermion α in lattice site i. The eigenvalues of $\Psi(i)$ are integers:

$$F(i) = 1, 2, ..., 4N$$

A list of these eigenvalues in every position of the lattice, at every time t, $n(t)$ will be a complete characterization of the fermiom number in the universe Φ.

$$n(t) = (F(1), F(2)), ..., F(L))(t)$$

$$\Phi = (|t\rangle\,, n(t)) \tag{3.9}$$

The state vector $|t\rangle$ evolves according with the Schrödinger equation

$$\frac{d}{dt}\,|t\rangle = -\frac{i}{\hbar} H\,|t\rangle \tag{3.10}$$

while for $n(t)$ Bell suggests a stochastic evolution. Let us define three tensors T_{nm}, J_{nm}, D_m:

$$J_{nm} = \sum_{qp} 2\Re\,\langle t|nq\rangle\,\langle nq| - \frac{i}{\hbar} H\,|mp\rangle\,\langle mp|t\rangle \tag{3.11}$$

$$D_m = \sum_{q} |\,\langle mq|t\rangle\,|^2 \tag{3.12}$$

and if $J_{mn} > 0$

$$T_{nm} = \frac{J_{nm}}{D_m} \tag{3.13}$$

otherwise $T_{nm} = 0$.

In an infinitesimal time interval dt a specific configuration m transmutes into one n with transition probability

$$dtT_{nm} \tag{3.14}$$

Therefore the change of a probability distribution P_n over configurations n is dictated by

$$\frac{dP_n}{dt} = \sum_m (T_{nm}P_m - T_{mn}P_n) \tag{3.15}$$

(3.15) is similar to the time derivative of (3.13):

$$\frac{dD_n}{dt} = \sum_m J_{nm} = \sum_m (T_{nm}D_m - T_{mn}D_n) \tag{3.16}$$

Since the solution of this differential equation must be unique, assuming that at some given time t_0

$$P_n(t_0) = D_n(t_0) \tag{3.17}$$

then (3.16) tells us that the solution of (3.15) is

$$P_n(t) = D_n(t) \tag{3.18}$$

The stochastic transition probabilities (3.14) are the Dirac field equivalent of the deterministic pilot wave equation (3.7).
Remember that we introduced a stochastic time evolution after we applied a discretization of space. This was only an assumption, there was nothing that pushed us to do so. As is the case that in a continuous limit stochastics may be eliminated from the formalism. Finally, the set of equations (3.8)-(3.18) is telling us that the fact of the matter is that at the time of creation God chose a set composed of a state vector $|0\rangle$, a fermion arrangement $n(0)$, and a probability distribution $D(0)$ assigned to the already selected state[2]. The whole set consequently evolved following (3.10) and (3.14).

3.2 Quantum equilibrium and non-equilibrium

The choice of a specific probability distribution is crucial if we require that Bohmian theory must have a predictive power at least equal to that owned by the Copenhagen interpretation. Precisely, the orthodox theory includes the Born rule as one of its main axioms

$$\rho(\vec{x}, t) = |\psi(\vec{x}, t)|^2 \tag{3.19}$$

As shown by Colin and Valentini[30], Bohm included it a posteriori claiming that at a given instant both the distribution of particles respects (3.19) and their momenta follow (3.5). In their paper they also calculated the trajectories for a simple harmonic oscillator

[2]This may seem a spooky and unnecessary demand: since the state has already been identified what is the meaning of defining a probability distribution? Nevertheless, states linked to different $D(0)$ have different time evolutions.

and for a hydrogen atom pointing out that even the slightest perturbation from (3.5) in the initial state leads to results different from standard quantum mechanics. As a matter of fact endowing a particle energy eigenstate with a very small amount of momentum can transform a bound state into a unbound one.

Observation 10. *If the universe millions of years ago had a momentum field different from the one prescripted in* (3.5) *we would not expect it to have nowdays particles with the measured quantum behaviours.*

If anything, if there is a difference in the probability distribution from the one dictated by the Born rule we expect the system to *relax* to quantum-equilibrium, as illustrated through numerical calculations by Towler, Russel and Valentini[31].

Observation 11. *The quantum-equilibrium is therefore a stable point in the space of probability distributions. As a result if the universe ever had a probability distribution different from the quantum-equilibrium it would have by now relaxed to it.*

3.3 How to improve the formalism

In my honest opinion the De Broglie-Bohm theory provides a more intuitive interpretation of the dynamical behaviour of the microscopic world. The formalism is also embedded with an innate non separability which is in perfect agreement with the arguments presented in the previous chapters. Nevertheless, the whole apparatus is not more robust than the orthodox one. Instead, even though apparently well justified, it is based on Copenhagen interpretation through the employment of Schrödinger equation. True, it gives to the latter a complete different sense, but the *condition of use* (the attribution of a *probability* distribution with a fuzzy meaning to an already defined state) could be just another, perhaps less restrictive, way to absorb the intrinsic statistical features of the orthodox theory. Less restrictive because the Born rule is not anymore a postulate, and hence a perturbation from the stable quantum equilibrium may allow the happening of new events that would be otherwise unpredictable using the prescriptions of Copenhagen's. However, a theory is as sound as its foundations. And the basement of Bohm theory, as it is today intended, can not provide the balance requested of a theory that should replace the common archetype.

A way to improve the formalism should be focusing on what is different from the orthodox theory on a more concrete level, the presence of a hidden variable, and thus trying to derive all the rest following an independent path. Hopefully the journey of research will shed light on our doubts and will show us if the De Broglie-Bohm theory can evolve into one which describes reality with more completeness, into another that is still equivalent to Copenaghen's or for last into yet another theory that will be proved to be utterly wrong.

Conclusions

After careful appraisal and study of the quoted sources I stressed the weaknesses of the Copenhagen interpretation:

- the paradoxical superposition of states and the intrinsic statistical nature of a system
- the problem of measurement and the state evolution by collapse
- the issue about determining what is an obeserver and what is observed

The solution to the previous issues resides in a more fundamental fact: if we want Quantum Mechanics to be a coherent and robust description of reality we have to clarify the ontology of the theory and rub the fuzziness off its postulates.

In order to do so I drew up a list of reasonable epistemological features that a sound theory of the physical world ought to possess:

1. it has to be consistent
2. it has to provide previsions in compliance with experimental evidence
3. it has to be falsifiable
4. it has to suffice Occam's razor
5. it has to abide by a ‘‘rule’’ of good sense

Armed with this vademecum I pointed out that the current leading version of Quantum Mechanics, through its postulates, while attempting to be more general as possible, is subtly assigning to reality the unrequested possibility of being more things at the same time. The latter endows the states with an intrinsic statistiscal trait which has to be removed or, better, replaced by a *non-postulated* additional structure[32].

The structure is provided by the existence of some hidden variables that would render the quantum world a statistical ensemble averaged over these individually unknown properties. Picking up the trail of Bell, I explained why this more intuitive approach was not commonly followed. It was due to a misunderstanding in an argument presented by Von Neumann where the author claimed that it was impossible to reformulate quantum mechanics in terms of hidden variables. In his famous work Bell showed, through the violation of the inequalities that now are named after him, that what really is impossible to achieve is the creation of a *local* theory of hidden variables. More specifically I outlined in detail how the inequalities are applied to the EPR paradox and to a Stern-Gerlach experiment, displaying their manifest violation.

Therefore I briefly discussed entanglement and faster-than-light signaling, both charging the ‘‘No-communication theorem” for its factual impossibility, both mentioning Valentini’s suggestion about its realization exploiting a non quantum equilibrium configuration of the universe. Valentini claims that the Born rule should not be a postulate of Quantum Mechanics and that, in principle, the probability density of finding a particle in a given place could not forcingly be equal to the squared modulus of the wave function.

Later I introduced the concepts of holism and nonseparability, to be intended as the inability to express a state as the direct product of its discrete constituents.
In particular I adumbrated the Aharonov-Bohm effect and invoked it to infer a non spacial separability concerning the electromagnetic field, which would violate Bell’s inequalities without implying the loss of Lorentz invariance.

In the last chapter I elucubrated a non-separable hidden variable formalism of quantum mechanics: the pilot wave theory by De Broglie-Bohm. In this framework a system of particles is completely described by their position *plus* a probability distribution assigned to the whole universe that influences the particle dynamics. Since, even though we can define a set of positions for the collection of particles, we can not physically measure them with extreme precision, this probability distribution represents the chance for the whole universe to *be* in that specific state (rather than to measure it in that state as in Copenhagen interpretation). As a matter of fact every single particle always *exists* in a spacial site and does not assume a position only throughout the act of measurement. The probability distribution must evolve as a wave equation so as to make the theory able to reproduce the results of quantum mechanics. The only wave equation which correctly replicates the previsions of the orthodox interpretation is the Schrödinger equation. Following these prescriptions one could calculate the trajectories for the particles composing a dynamical system. I considered the case of an electron in some hydrogenoid atom bound states and exhibited how Bohm’s theory tells us that it should be perfectly still. This counterintuitive result is in accordance with the fact that electrons in bound states do not irradiate.

In the end I expressed my critics about the model: despite the pilot wave theory better suffices the epistemological demands of the first section, it should derive the wave equation in a independent way to completely emancipate itself from the Copenhagen interpretation and be fully consistent.

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